NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
CELEBRATING 100 YEARS

Playing with Mathematical Ideas preK-grade2

#NCTM100

June 29 2020

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Playing with Mathematical Ideas in studio and outdoor spaces for all learners

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NCTM CENTENNIAL MEETING 2020

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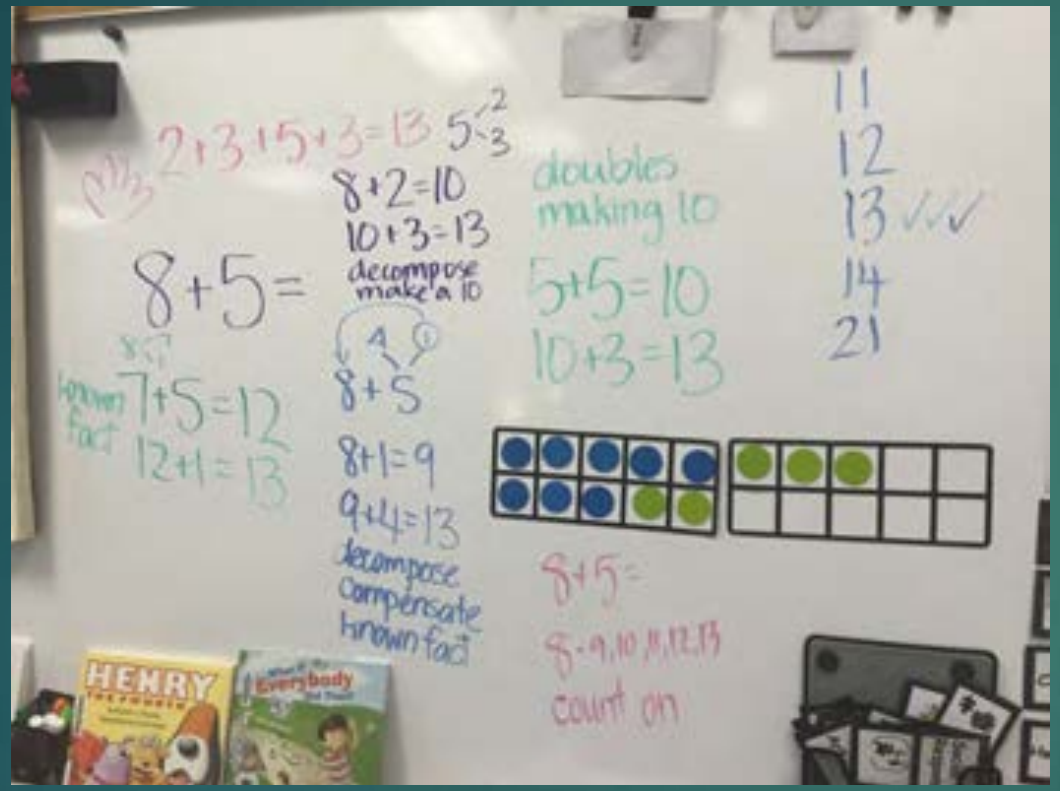
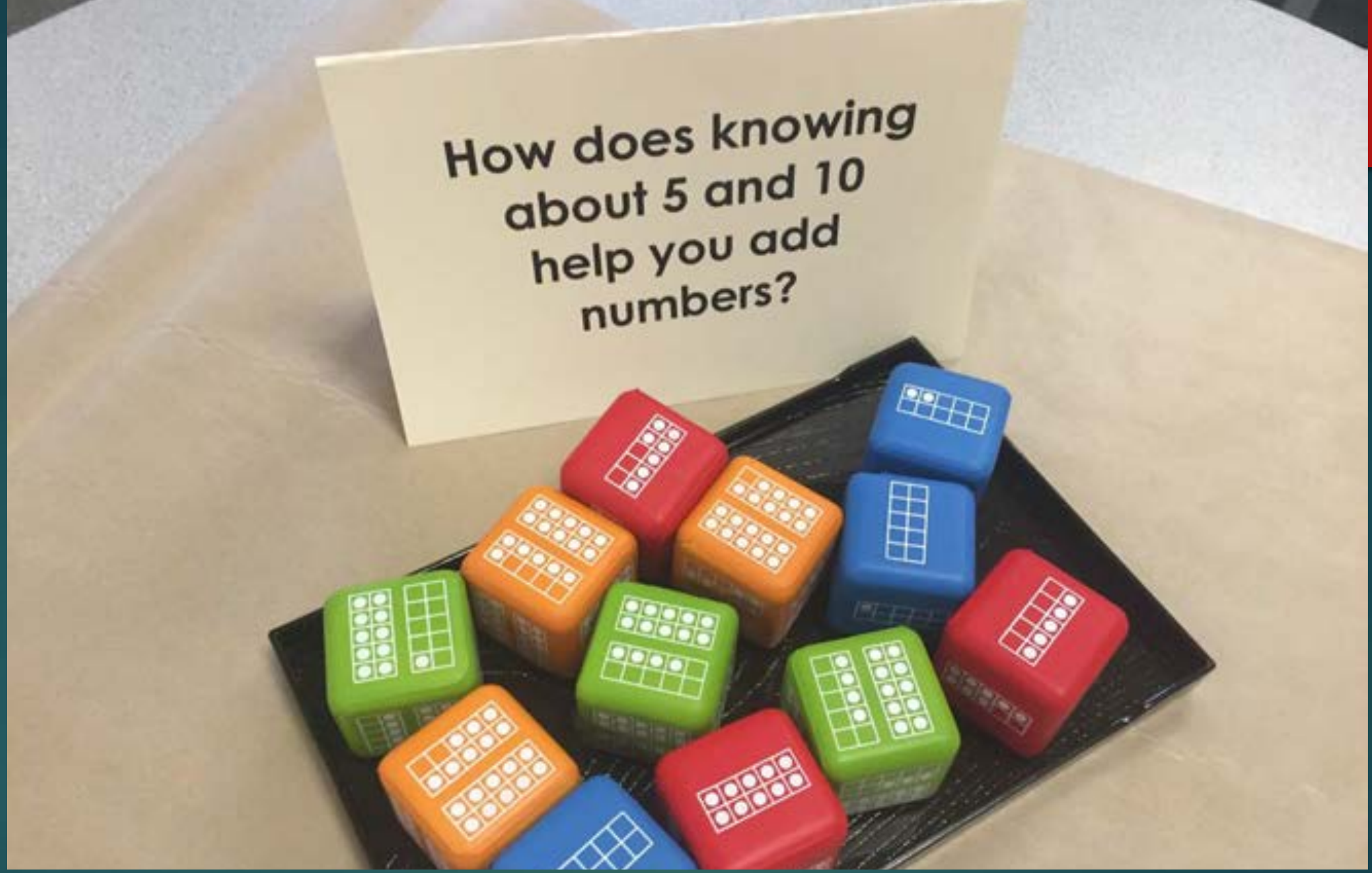
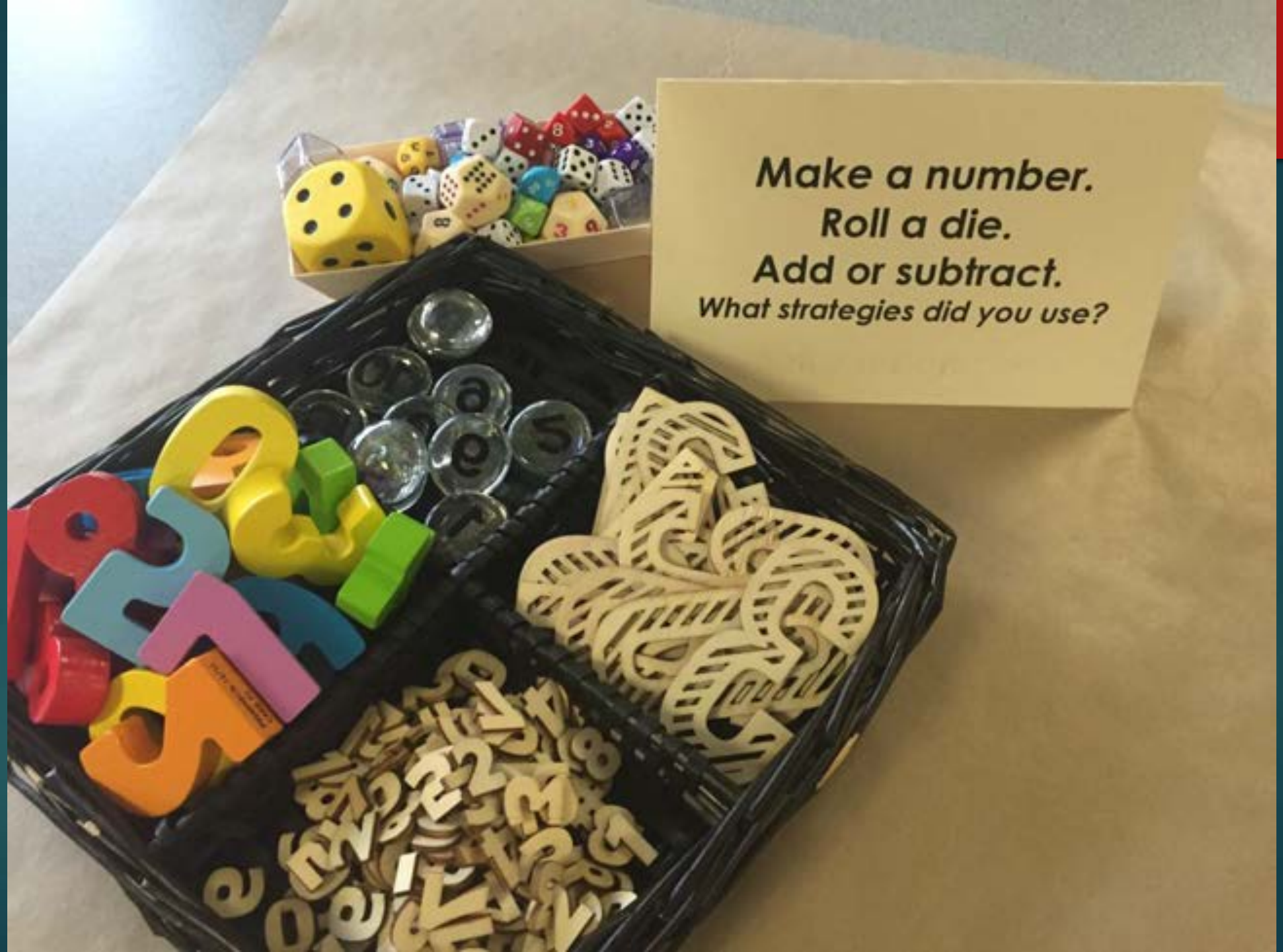
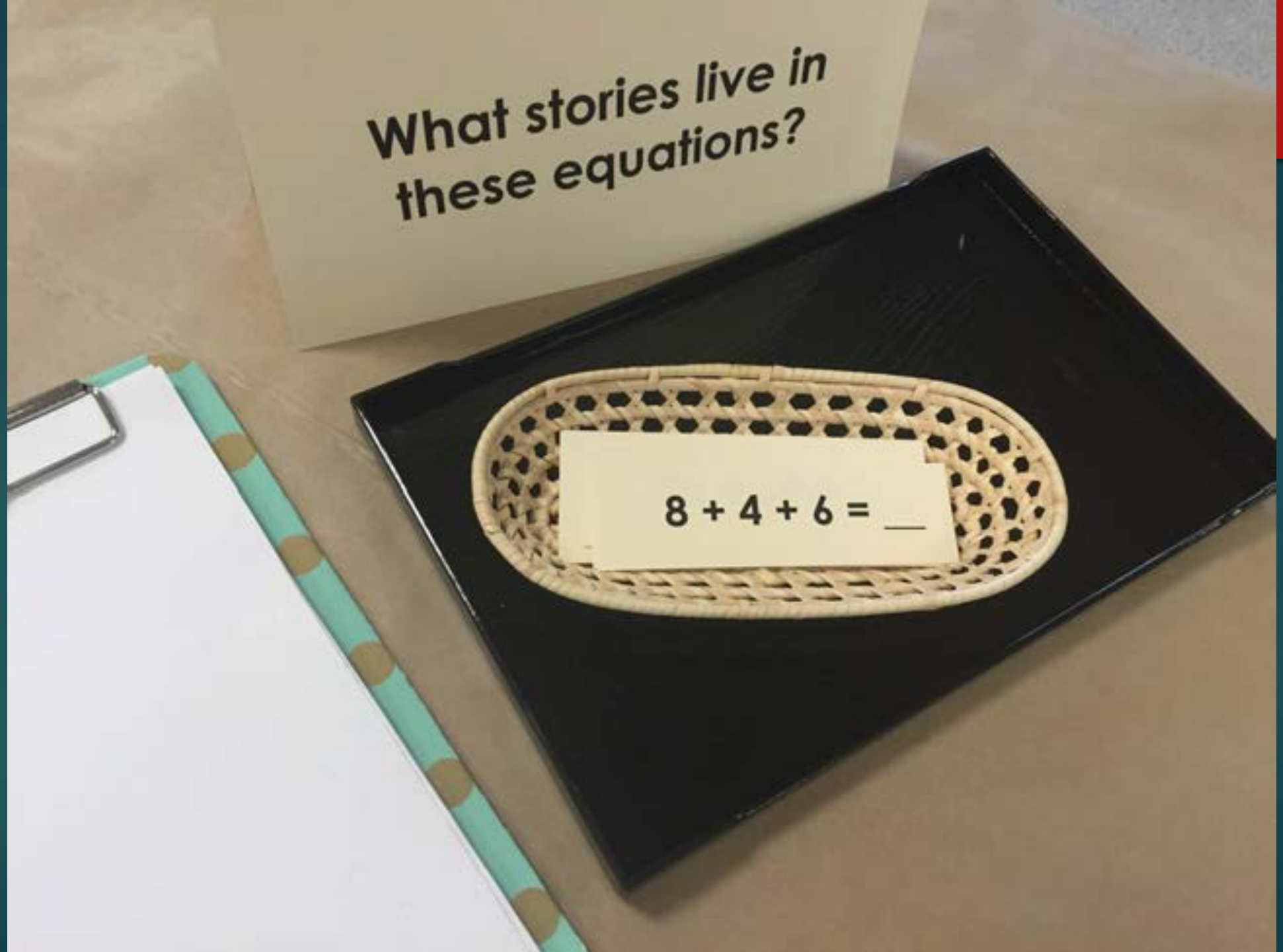
Land Acknowledgement

Investigating Addition and Subtraction Grades 1&2



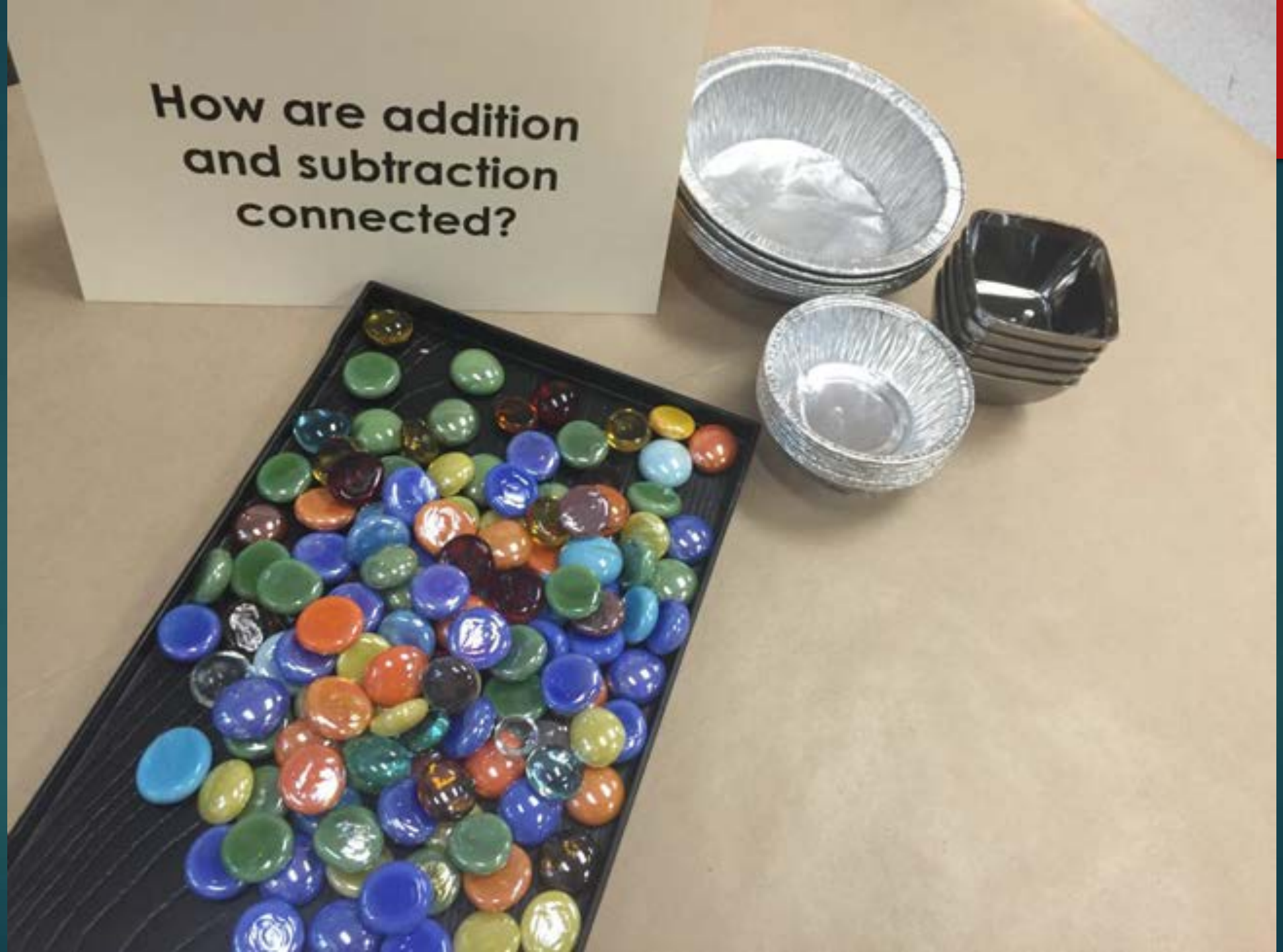

How does knowing about 5 and 10 help you add numbers?

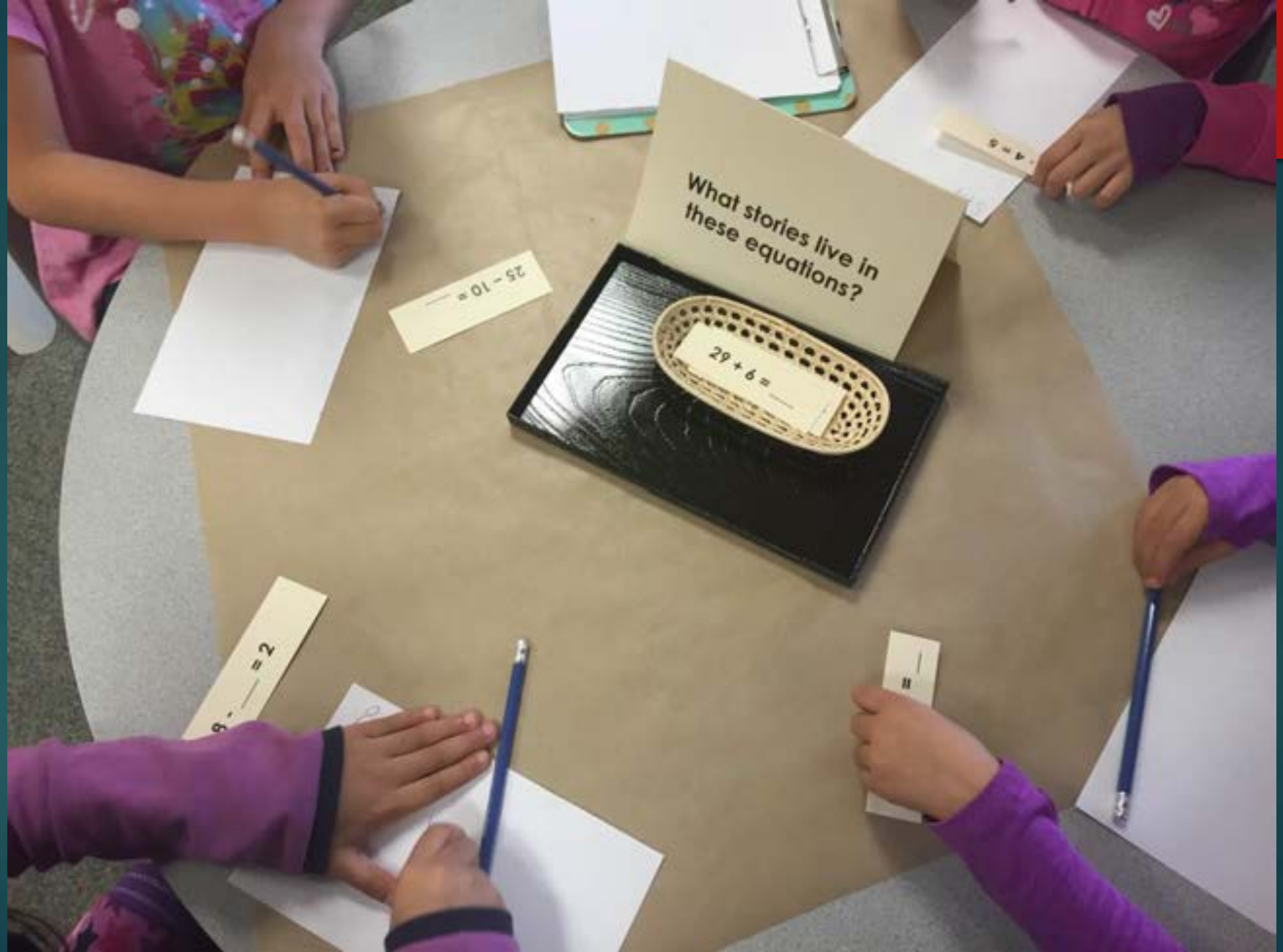

Make a number.
Roll a die.
Add or subtract.
What strategies did you use?


What stories live in these equations?
8 + 4 + 6 = __


What different ways
can you make 10?


How many different ways can you solve _____?
48 + 6 = ___
= 10


How are addition
and subtraction
connected?


What stories live in these equations?
29 + 6 = ___
25 − 10 = ___

Guiding Questions for The Studio

- *What does it mean to belong in a space?*
- *How is math connected to you and the world around you?*

In The Studio

"It's a math sun!
It's full of math!"

– Grauer Kindergarten student

Influences...

- Pedagogy of Play
  - Harvard Graduate School of Education
- Playful Inquiry
  - Opal School, Portland Oregon
- The Reggio Emilia Approach
  - Reggio Children, Reggio Emilia, Italy

What does it mean to play with ideas and concepts?

“Play is to mathematics what books are to reading.”

DAN FINKEL, MATHEMATICIAN, SEATTLE

MATHFORLOVE.COM

“the ball toss”

- Loris Malaguzzi of Reggio Emilia, Italy often spoke of the ball toss or the back and forth of a ping pong game as a metaphor for the roles of adults and children in teaching and learning.

questions to nurture mathematical thinking, play and inquiry:

- How are these alike?
  - How are these different?
- What do you notice?
  - What do you wonder?
- What do you think?
- How do you know?

Some frames...

- What stories live within...?
- What do you notice? What do you wonder?
- What is the relationship between....?
- What are the connections between...?
- What happens when...?
- How many different...?
- How do these materials help you think about...?
- Where do we see...in the world?

Connections

- math to self
- math to world
- math to math

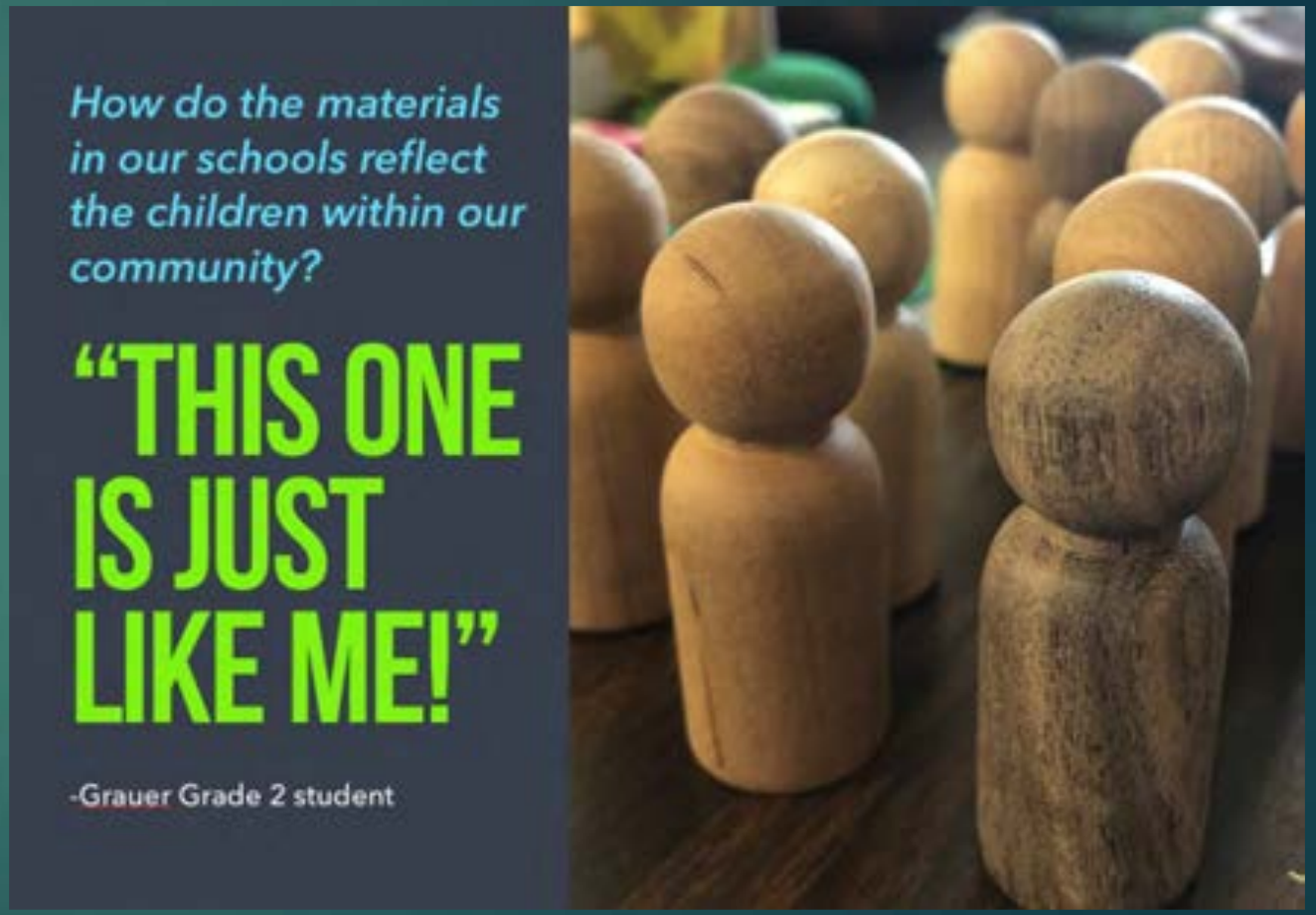
PLAY

- playing with materials
- playing with language
- playing with ideas

MIRRORS, WINDOWS & DOORS

- Dr. Rudine Sims Bishop
- Dr. Rochelle Gutierrez

  - *equity, access and re-humanizing mathematics*

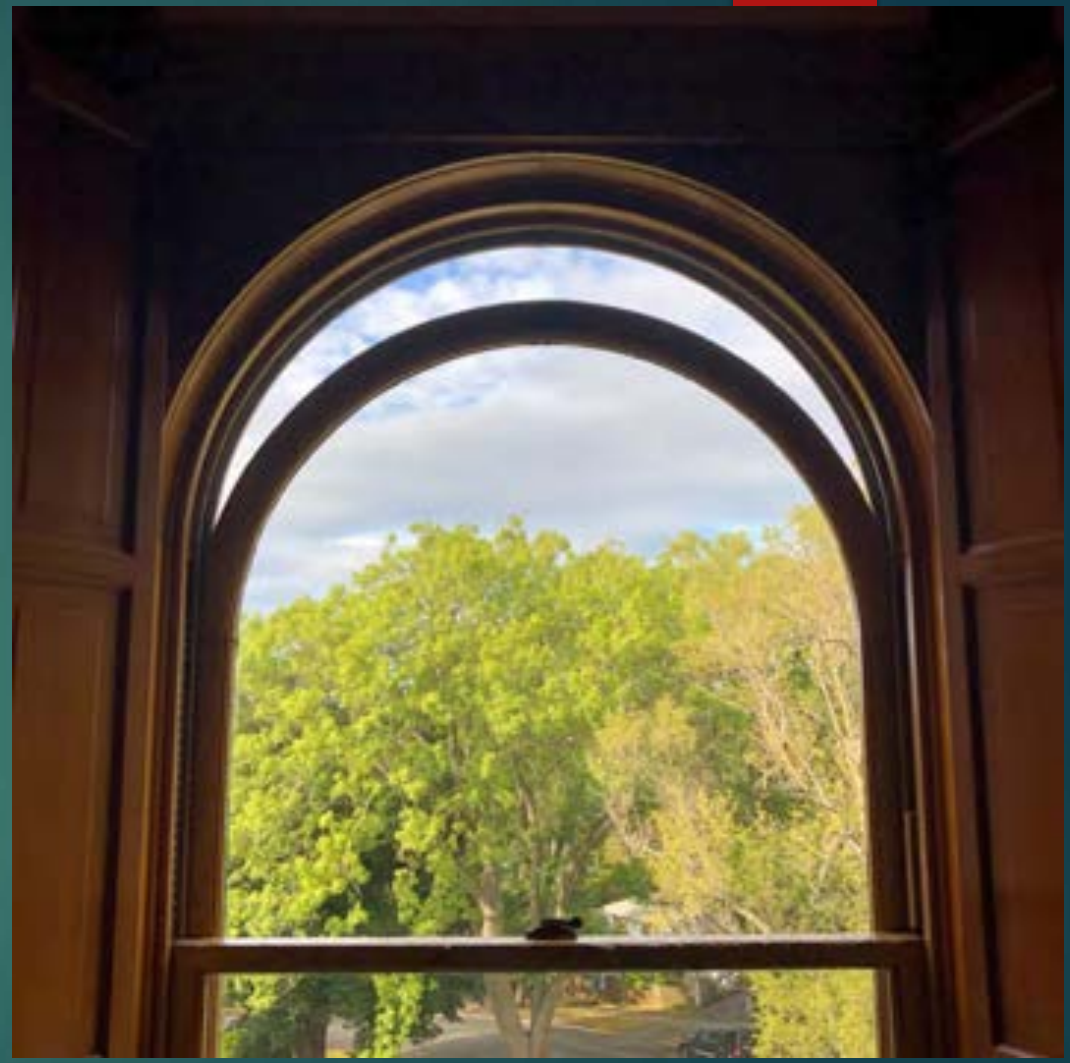


MIRRORS

- Do students see themselves reflected in your classroom mathematical experiences?

WINDOWS

- Do students have opportunities to be exposed to new perspectives and worldviews?



DOORS

- Do students have openings to new opportunities and to reach their goals?

in The Studio


What mathematics do you think about and do while you are designing and creating string art?


What a marshmallow brings

September 13 2018

How do art materials inspire children to play with mathematical ideas?

What authentic materials and tools can we use to enhance mathematical [illegible] and learning?

"My Grandpa has one of those."

-Grauer Grade 1 student

How do the materials in our schools reflect the children within our community?

"THIS ONE IS JUST LIKE ME!"

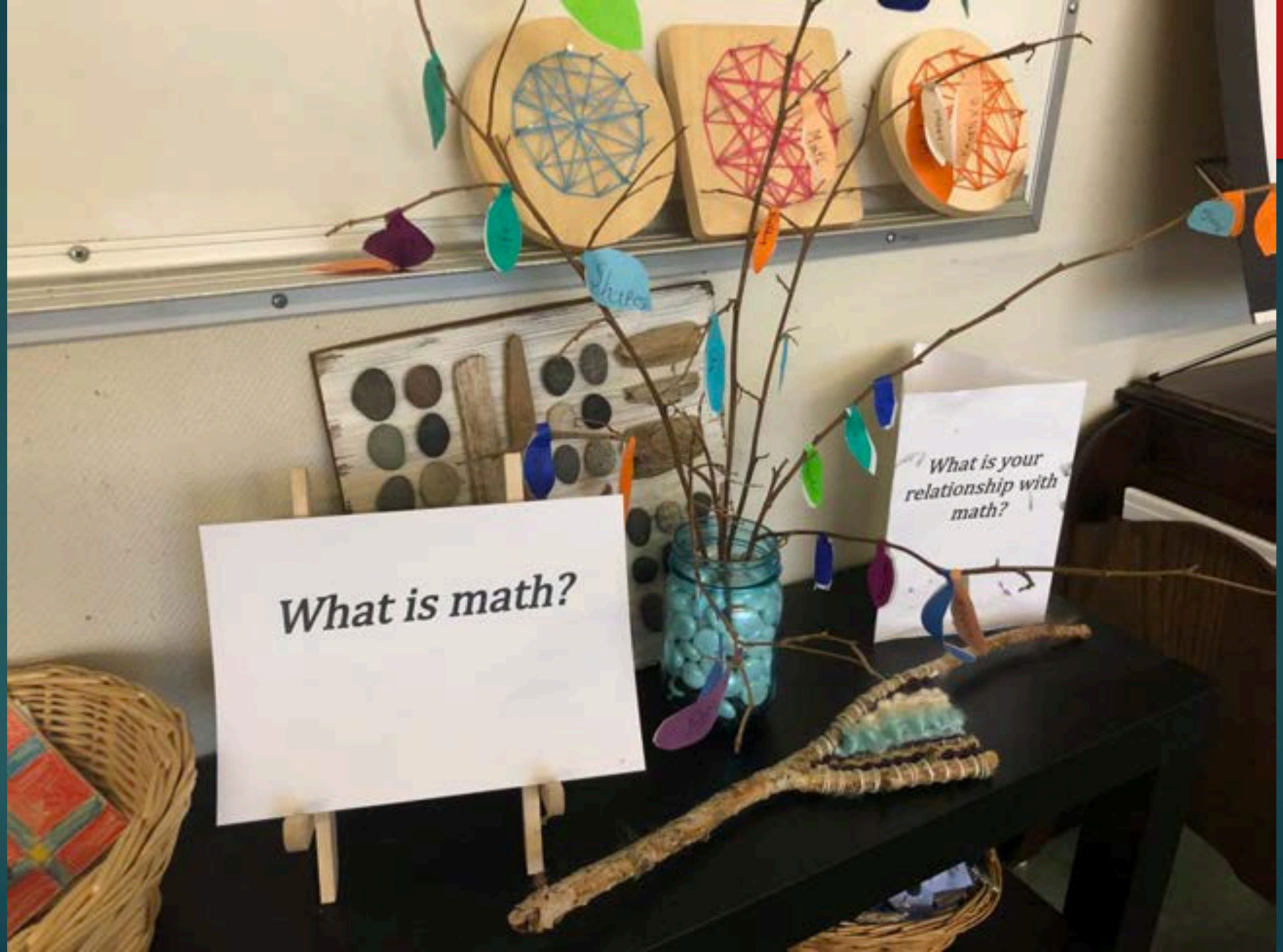
-Grauer Grade 2 student

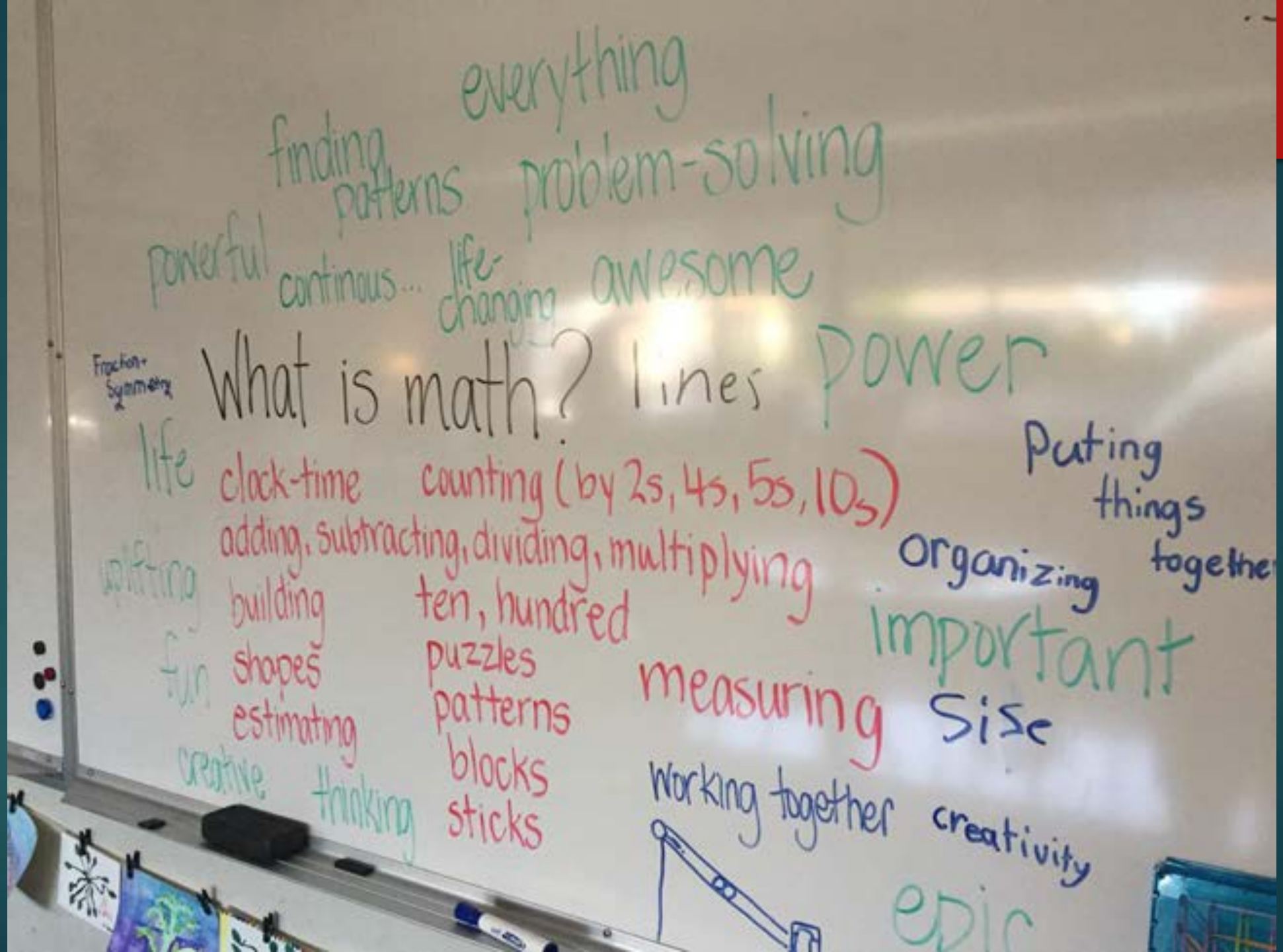
What materials bring children together to learn with each other?


which shape doesn't belong?
if...
TEN TIMES BETTER
MANDALA
THE COOKIE


What math do you see?


How are these shapes alike?
How are these shapes different?


What is math?
What is your relationship with math?

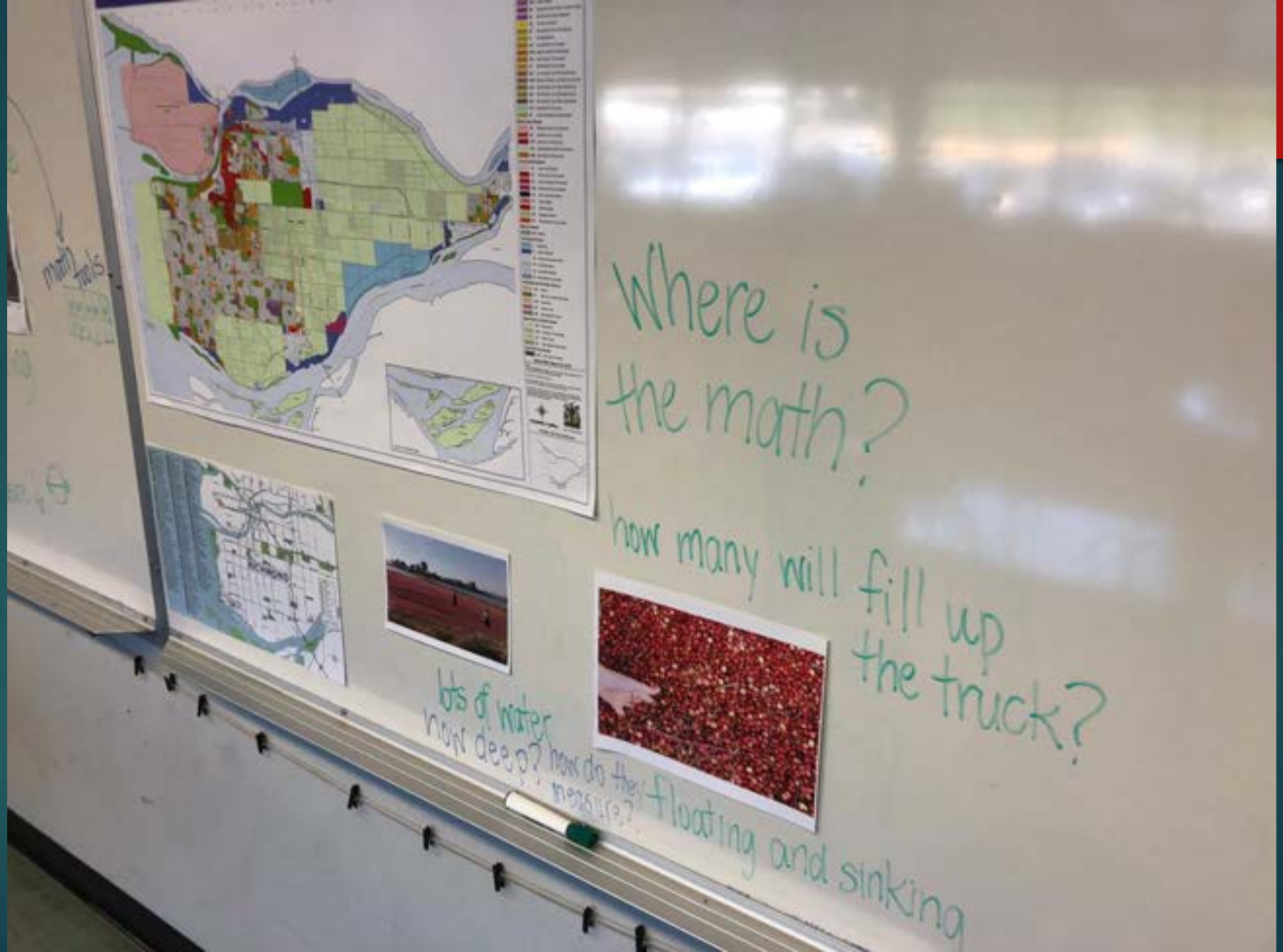

everything
finding patterns
problem-solving
powerful
continous...
life-changing
awesome
What is math?
lines
power
Fraction+ Symmetry
life
clock-time
counting (by 2s, 4s, 5s, 10s)
Puting things together
adding, subtracting, dividing, multiplying
organizing
building
ten, hundred
important
shapes
puzzles
measuring
Size
estimating
patterns
fun
blocks
working together
creativity
creative
thinking
sticks
epic

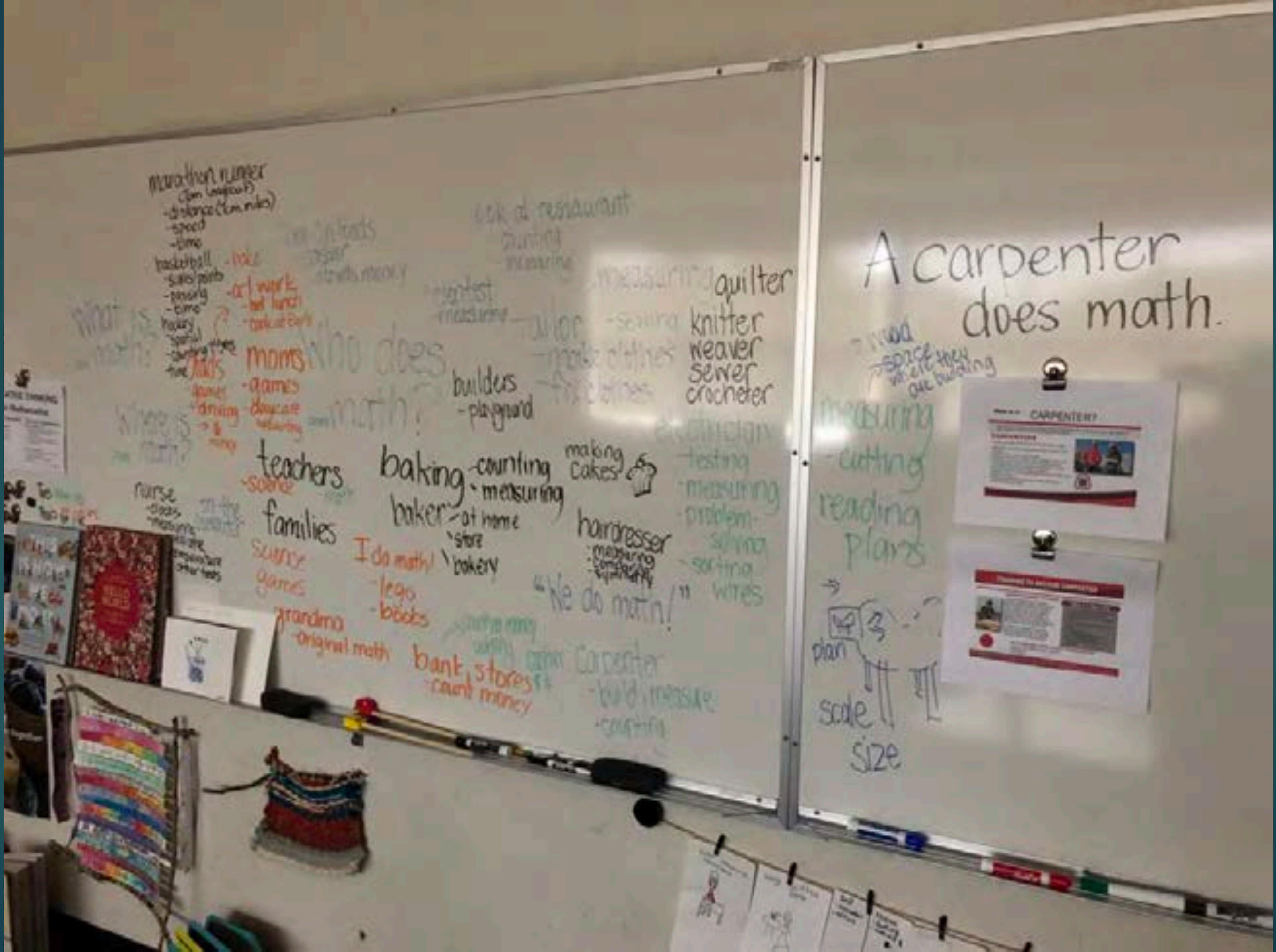

What is math?

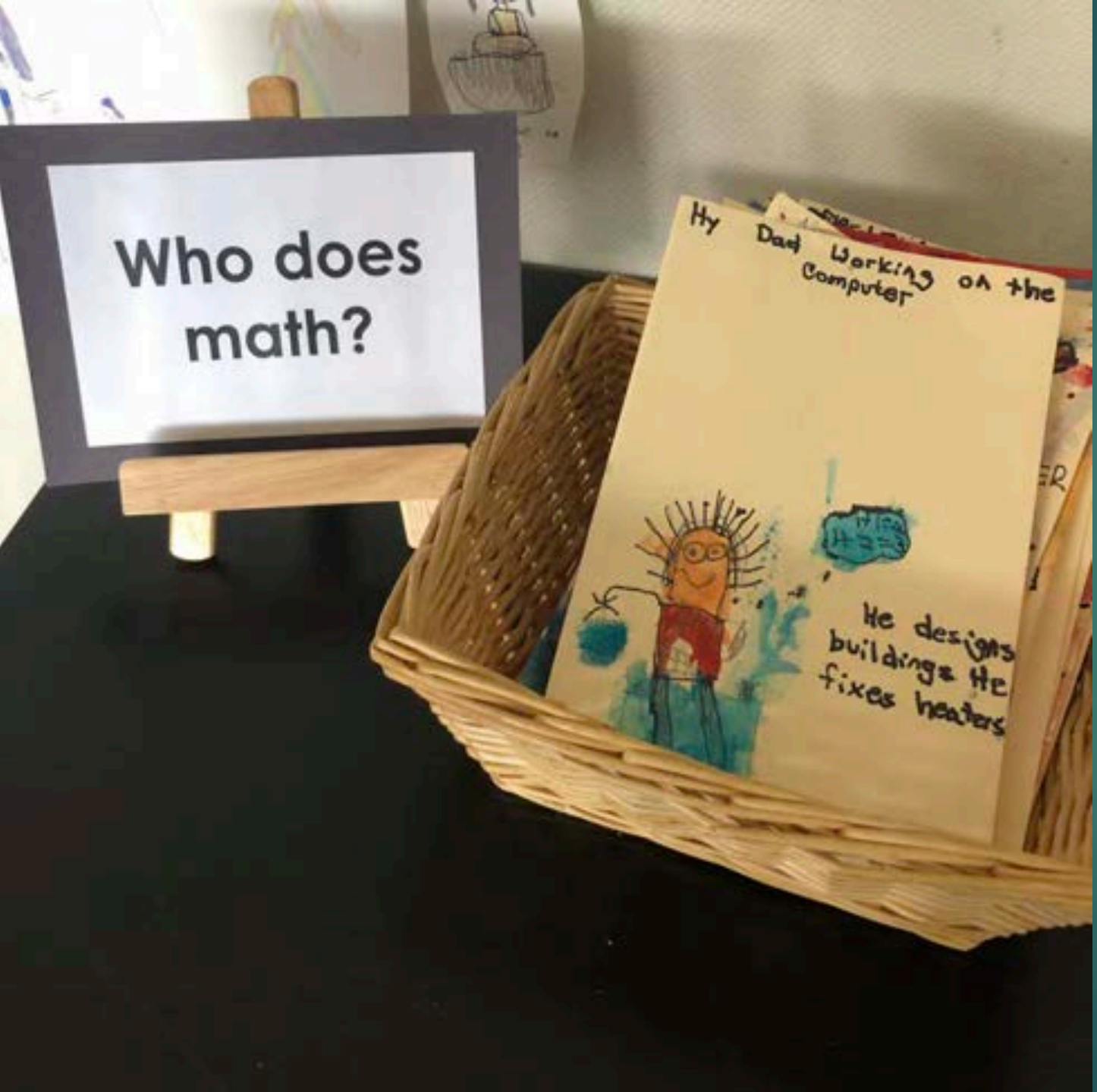

What is your
relationship with
math?

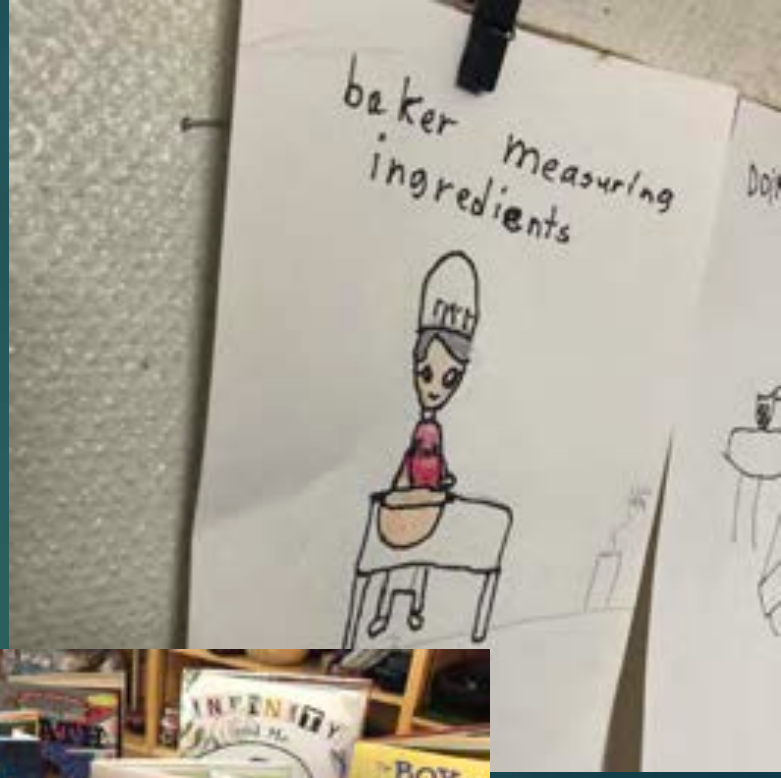

How does math
make you feel?
50
40
60
20

Where is the math?
Investigating Cranberries


Where is the math?
how many will fill up the truck?
lots of water
floating and sinking


A carpenter does math.
marathon runner
basketball
moms
-games
builders
-playground
quilter
knitter
weaver
sewer
crocheter
teachers
baking
-counting
-measuring
making cakes
families
baker
-at home
-store
-bakery
nurse
hairdresser
science
games
I do math!
lego
books
grandma
-original math
"We do math!"
bank
stores
-count money
Carpenter
-build, measure
-counting
measuring
-cutting
reading plans
plan
scale
size
CARPENTER?


Who does math?
My Dad Working on the Computer
He designs buildings He fixes heaters


baker measuring ingredients


Blockhead
Counting on KATHERINE
THE GIRL WITH A MIND FOR MATH
WE DO MATH
·TO THINK & LEARN
·IN OUR JOBS
·TO MAKE OUR WORLD BETTER
LEONARDO DA VINCI
Meet Elsie MacGill


HELLO WORLD
Hatem Aly
THE PROUDEST BLUE
Boxitects
Kim Smith
THIS IS HOW WE DO IT
MATT LAMOTHE
RAINBOW WEAVER
TEJEDORA DEL ARCOIRIS
Crescent Moons and Pointed Minarets
MEET Viola Desmond
THE GIRL WITH A MIND FOR MATH
The Story of Raye Montague
What If...
The Colors of Us
THE BOY WHO LOVED MATH
KATHERINE
Nothing Stopped Sophie
the world is not a rectangle
A BOOK IN FOUR LANGUAGES
MY NUMBERS
One Eagle Soaring

Vision for The Studio

- A space to re-imagine the teaching and learning of mathematics through an inclusive, flexible learning environment that offers choice of materials to consider mathematics through the aesthetic dimension.

Goals for The Studio Experience

- to disrupt thinking about mathematics teaching and learning
- to learn about the affordances of different materials
- to consider how the environment can support thinking and learning

for educators

- to learn more about instructional approaches that focus on deeper learning, inquiry and place-based and culturally responsive pedagogies
- to consider ways to teach mathematics more holistically, weaving together elements of our curriculum framework

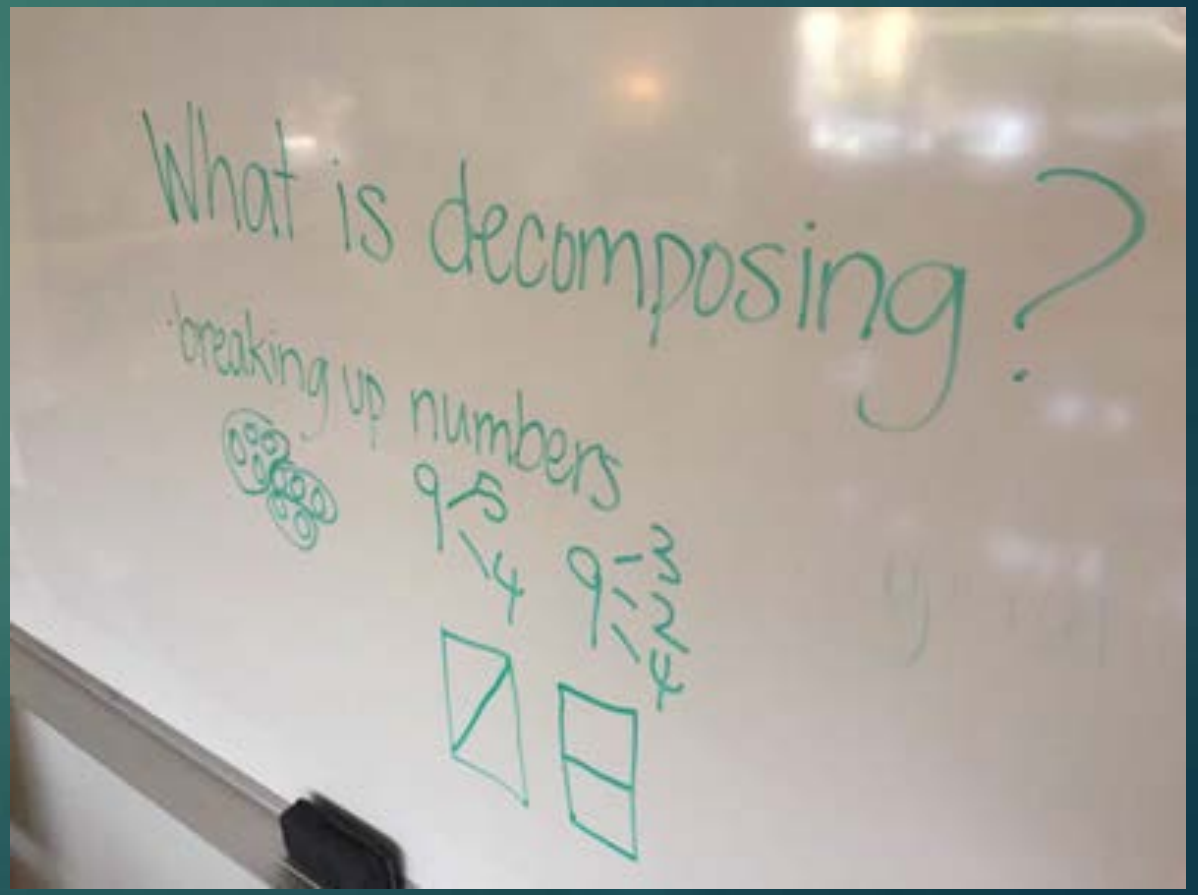
for students

- to have opportunities to experience math to math, math to self and math to world connections
- to have choice in where they choose to learn, the materials they use and how they share their thinking and learning and what projects they might take up

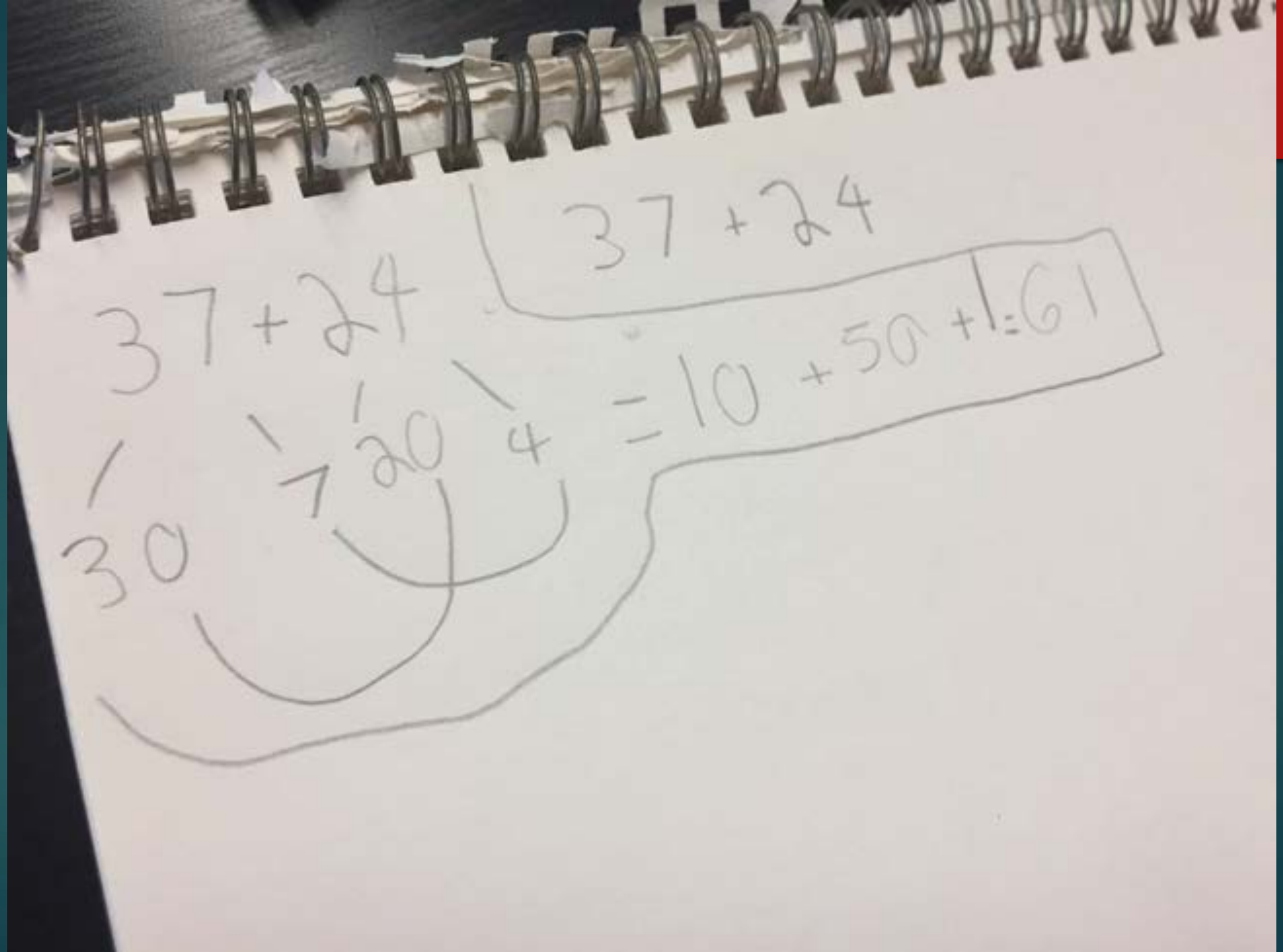
Connecting Big Ideas/Concepts

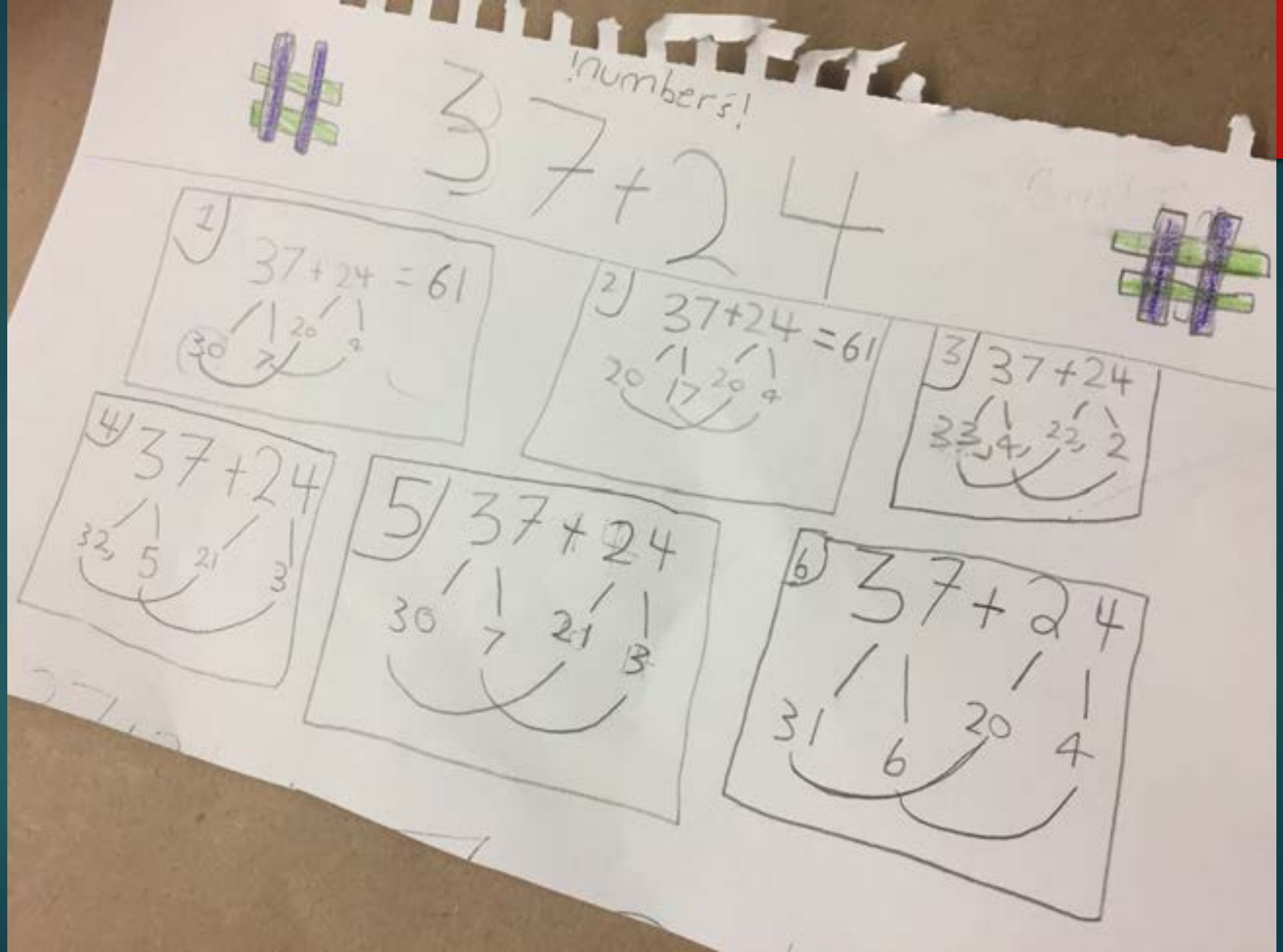
- Identity
- Story
- Place
- Community
- Relationship
- Connection
- Decomposing
- Change
- Transformation
- Equivalence
- Shape
- Pattern

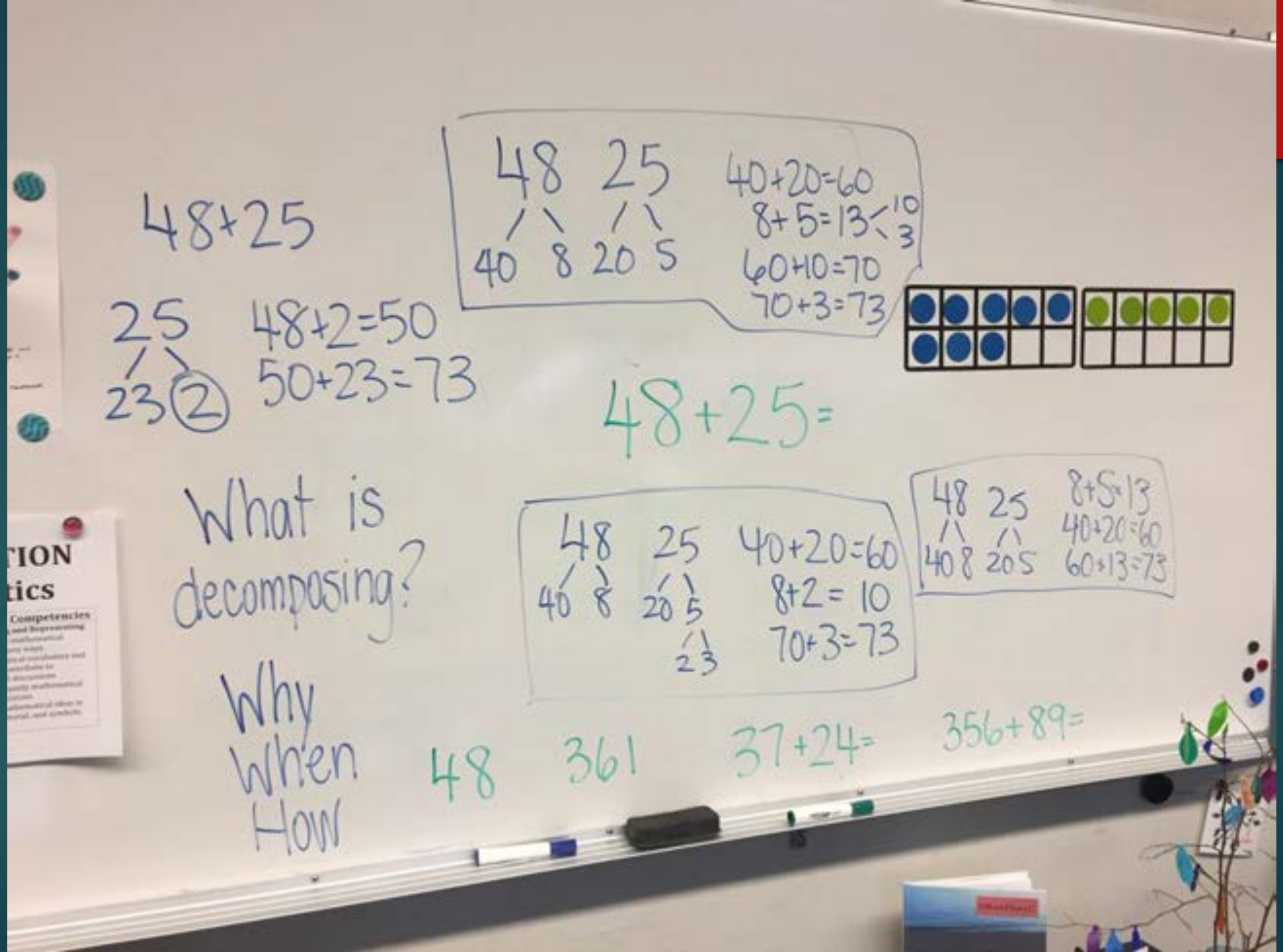
Teaching and Learning Through a Mathematical Big Idea/Concept: Decomposing




What is decomposing?
How might these materials help you think about decomposing quantities and shapes?


37 + 24
37 + 24
30
20
4
= 10 + 50 + 1 = 61


#numbers!
37+24
1) 37+24=61
30 7 20 4
2) 37+24=61
20 17 20 4
3) 37+24
4) 37+24
32 5 21 3
5) 37+24
30 7 21 3
6) 37+24
31 6 20 4

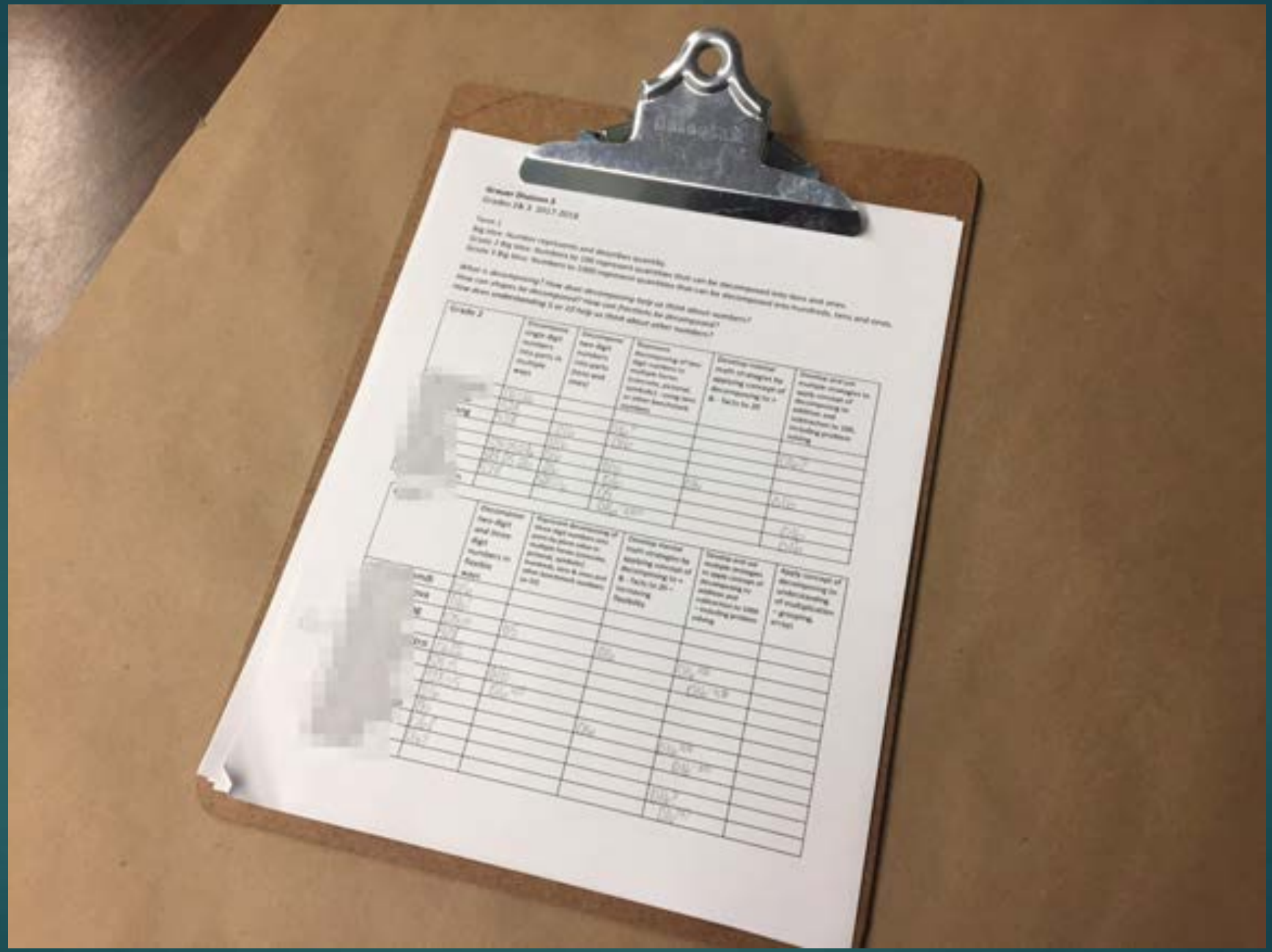

48+25
25
23 2
48+2=50
50+23=73
48 25
40 8 20 5
40+20=60
8+5=13
10
3
60+10=70
70+3=73
48+25=
What is decomposing?
Why
When
How
48 25
40 8 20 5
40+20=60
8+2=10
70+3=73
23
48 25
40 8
8+5=13
40+20=60
60+13=73
48
361
37+24=
356+89=


24000


48
20
20
8

Decomposing

1. Why do we use decomposing?
So you wouldnt panic.
2. When we use decomposing?
When you find math to hard.
How we use decomposing?
braking numbers a part.



Term 1

Big Idea: Number represents and describes quantity.

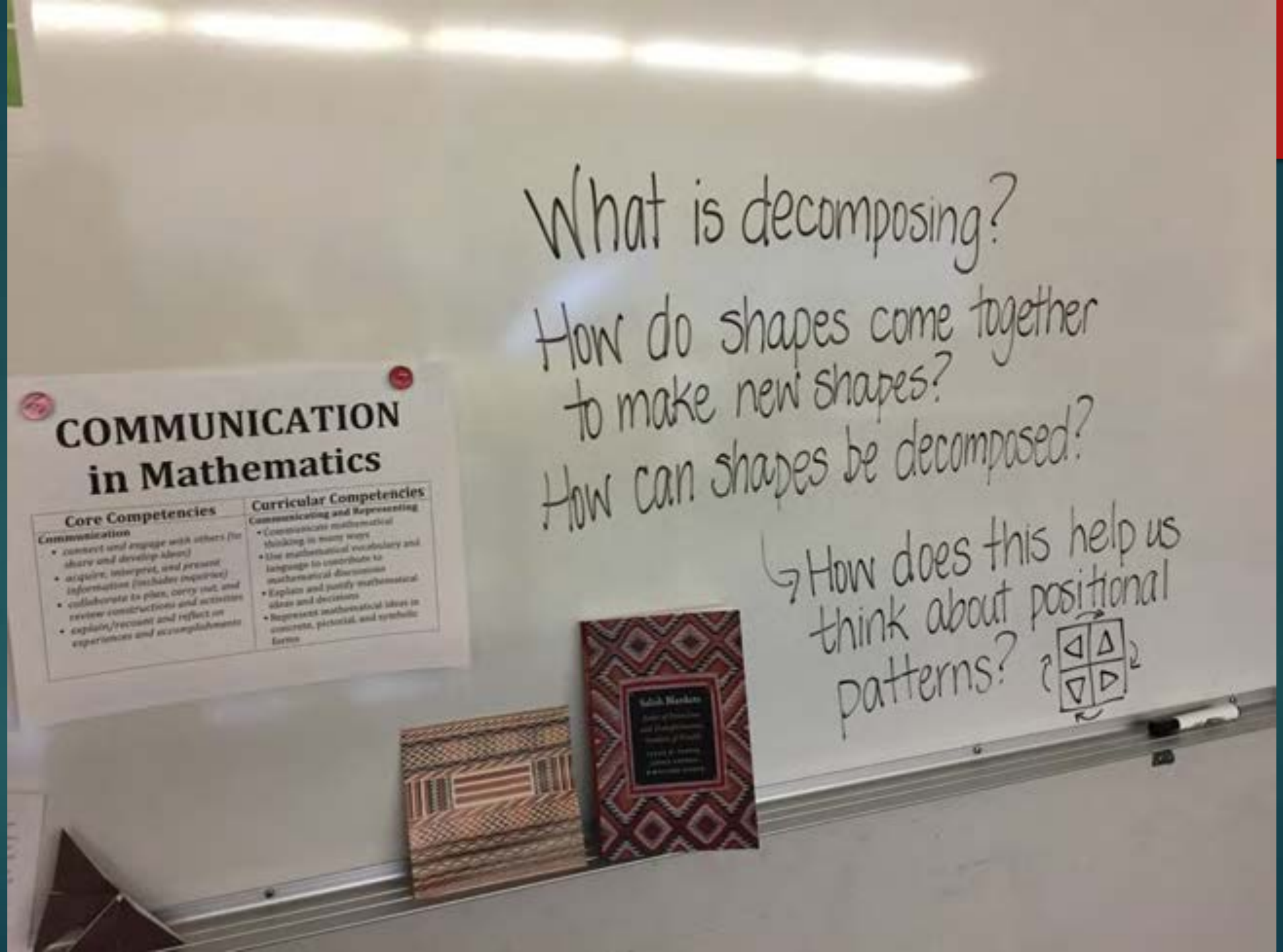
Grade 2 Big Idea: Numbers to 100 represent quantities that can be decomposed into tens and ones.

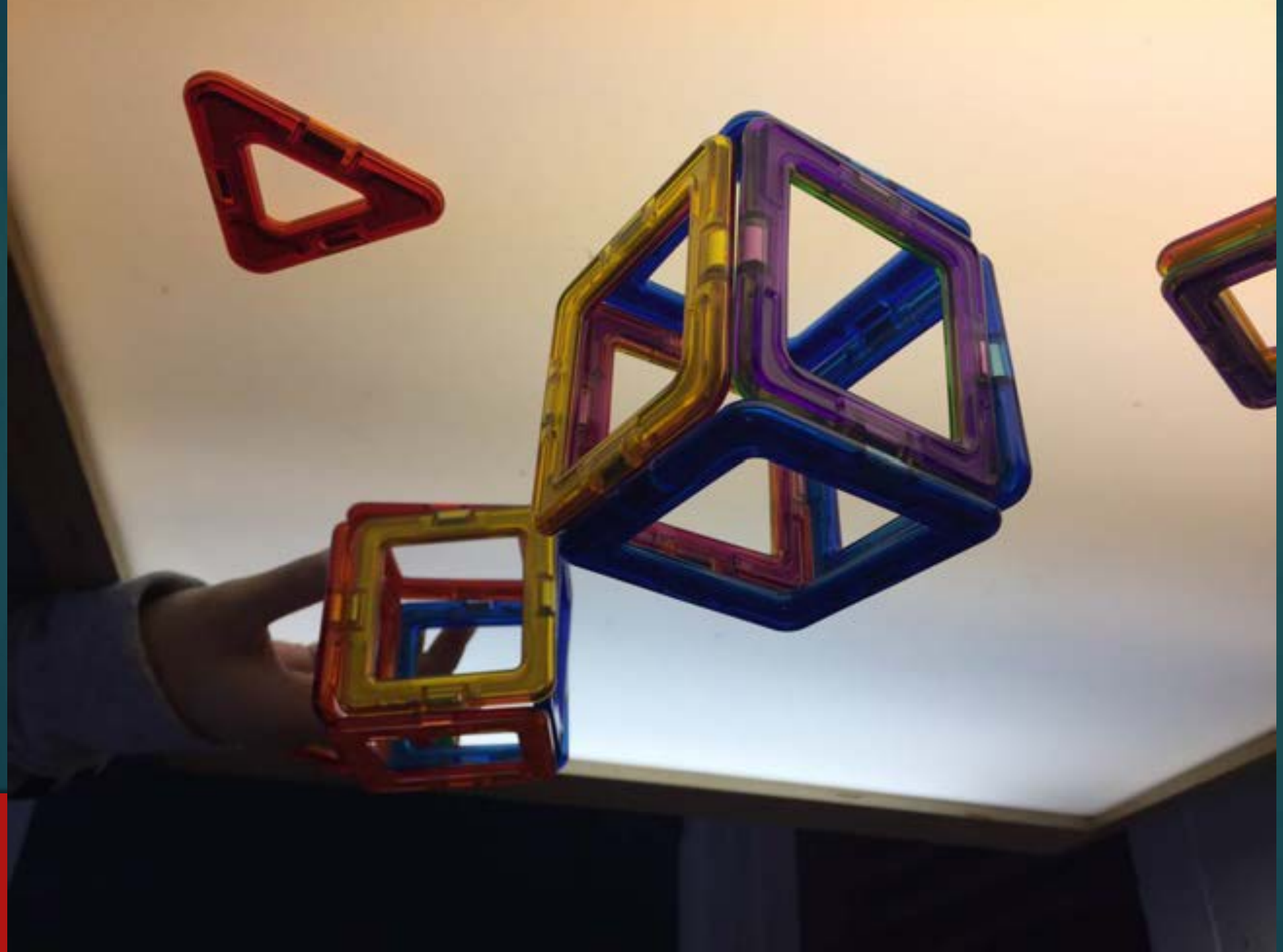
Grade 3 Big Idea: Numbers to 1000 represent quantities that can be decomposed into hundreds, tens and ones.

What is decomposing? How does decomposing help us think about numbers?
How can shapes be decomposed? How can fractions be decomposed?
How does understanding 5 or 10 help us think about other numbers?

Grade 2	Decompose single digit numbers into parts in multiple ways	Decompose two-digit numbers into parts (tens and ones)	Represent decomposing of two-digit numbers in multiple forms (concrete, pictorial, symbolic) - using tens or other benchmark numbers	Develop mental math strategies by applying concept of decomposing to + & - facts to 20	Develop and use multiple strategies to apply concept of decomposing to addition and subtraction to 100, including problem solving
[redacted]ista	05 016				
[redacted]l	528		016		016?
[redacted]g	528	016	016		
[redacted]		016			
[redacted]	528 05 016	016	016	016	016
[redacted]	528 05 016	016	016		
[redacted]	528	528 016	05		016
[redacted]hin			016 - 1000		016

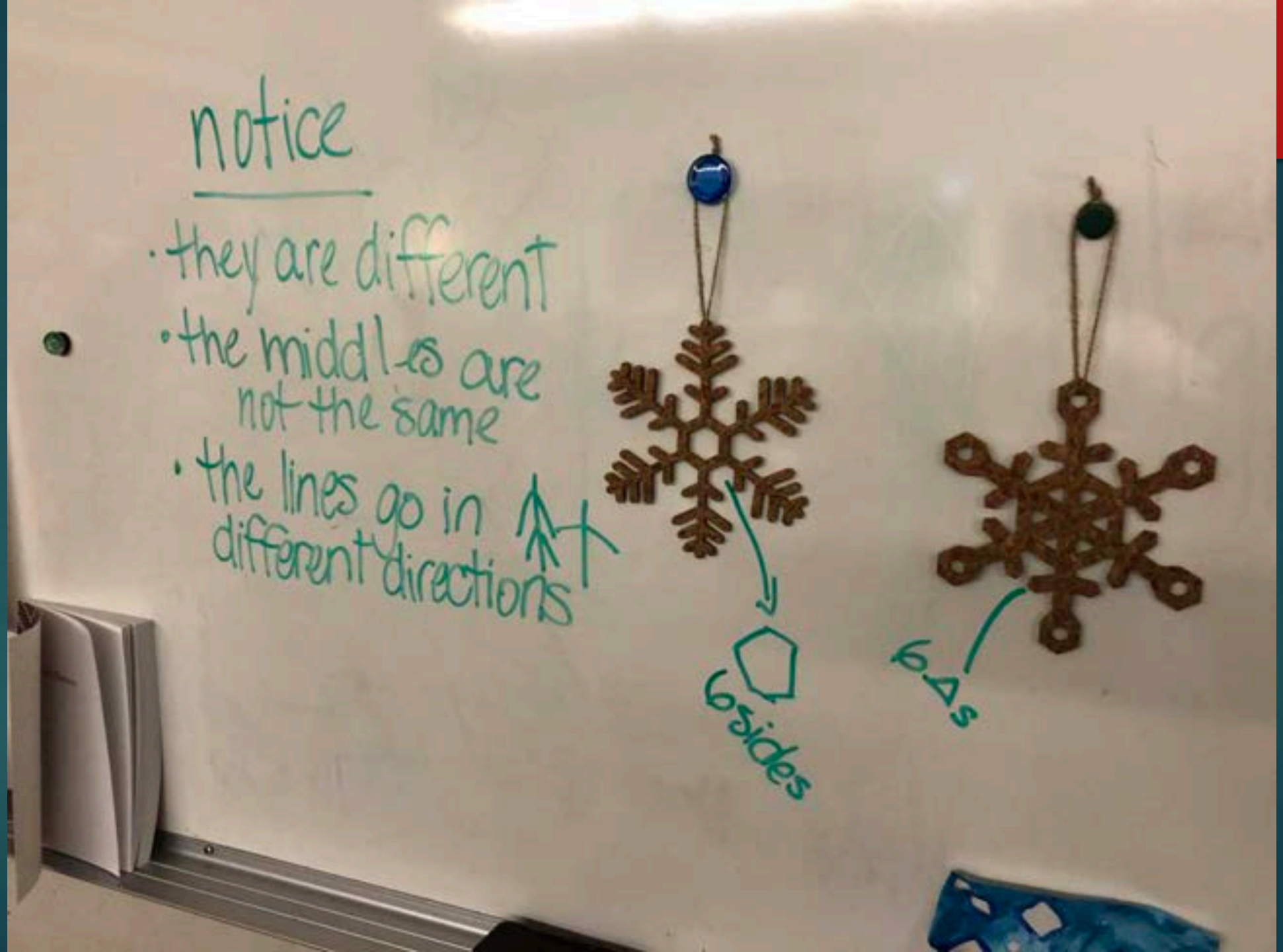
Grade 3	Decompose two-digit and three-digit numbers in	Represent decomposing of three-digit numbers into parts by place value in multiple forms (concrete, pictorial, symbolic) - hundreds, tens & ones and other benchmark numbers	Develop mental math strategies by applying concept of decomposing to + & - facts to 20 – increasing	Develop and use multiple strategies to apply concept of decomposing to addition and subtraction to 1000 – including problem	Apply concept of decomposing to understanding of multiplication – grouping, arrays

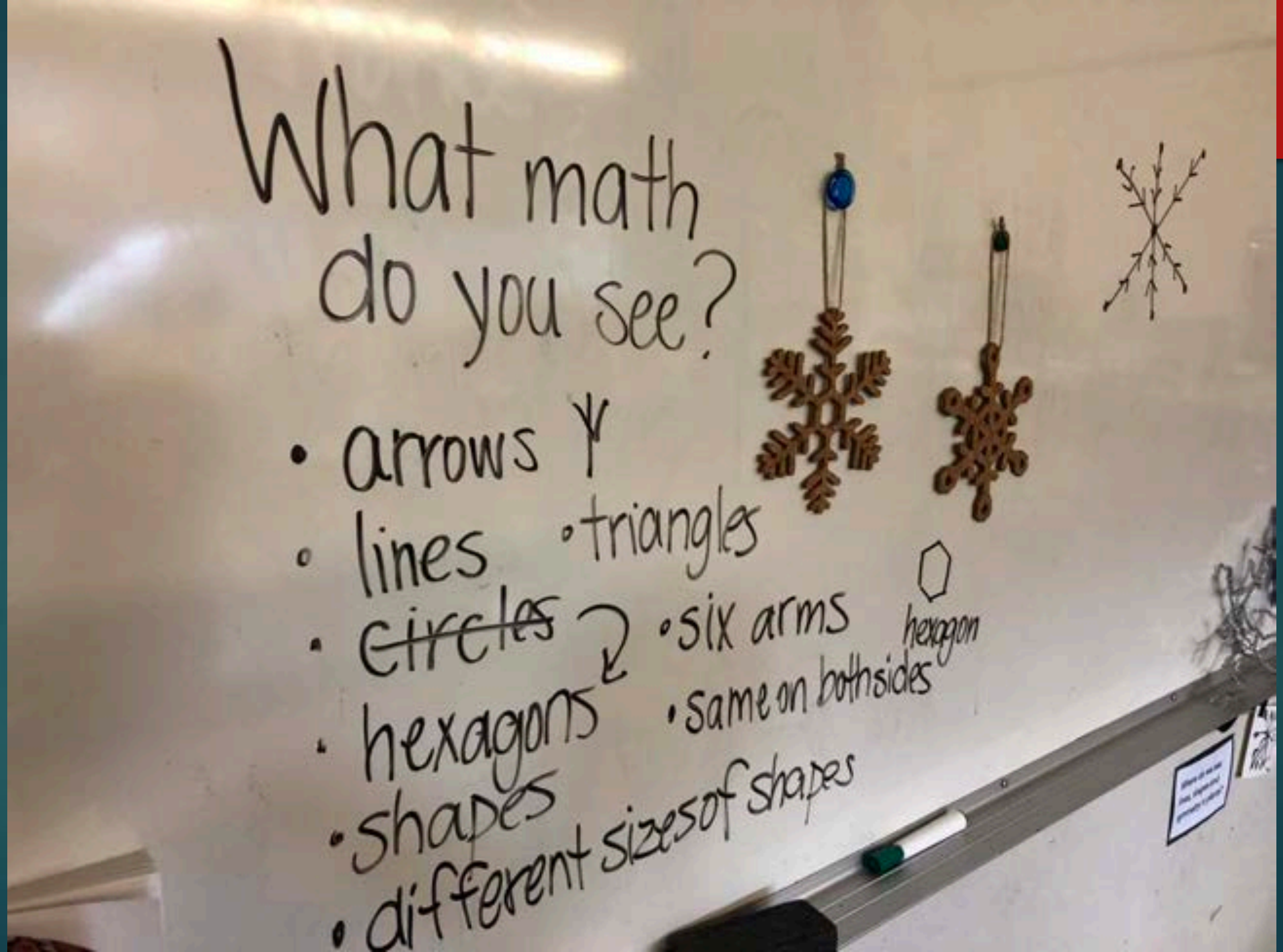

What is decomposing?
How do shapes come together to make new shapes?
How can shapes be decomposed?
How does this help us think about positional patterns?
COMMUNICATION in Mathematics
Core Competencies
Communication
Curricular Competencies
Communicating and Representing

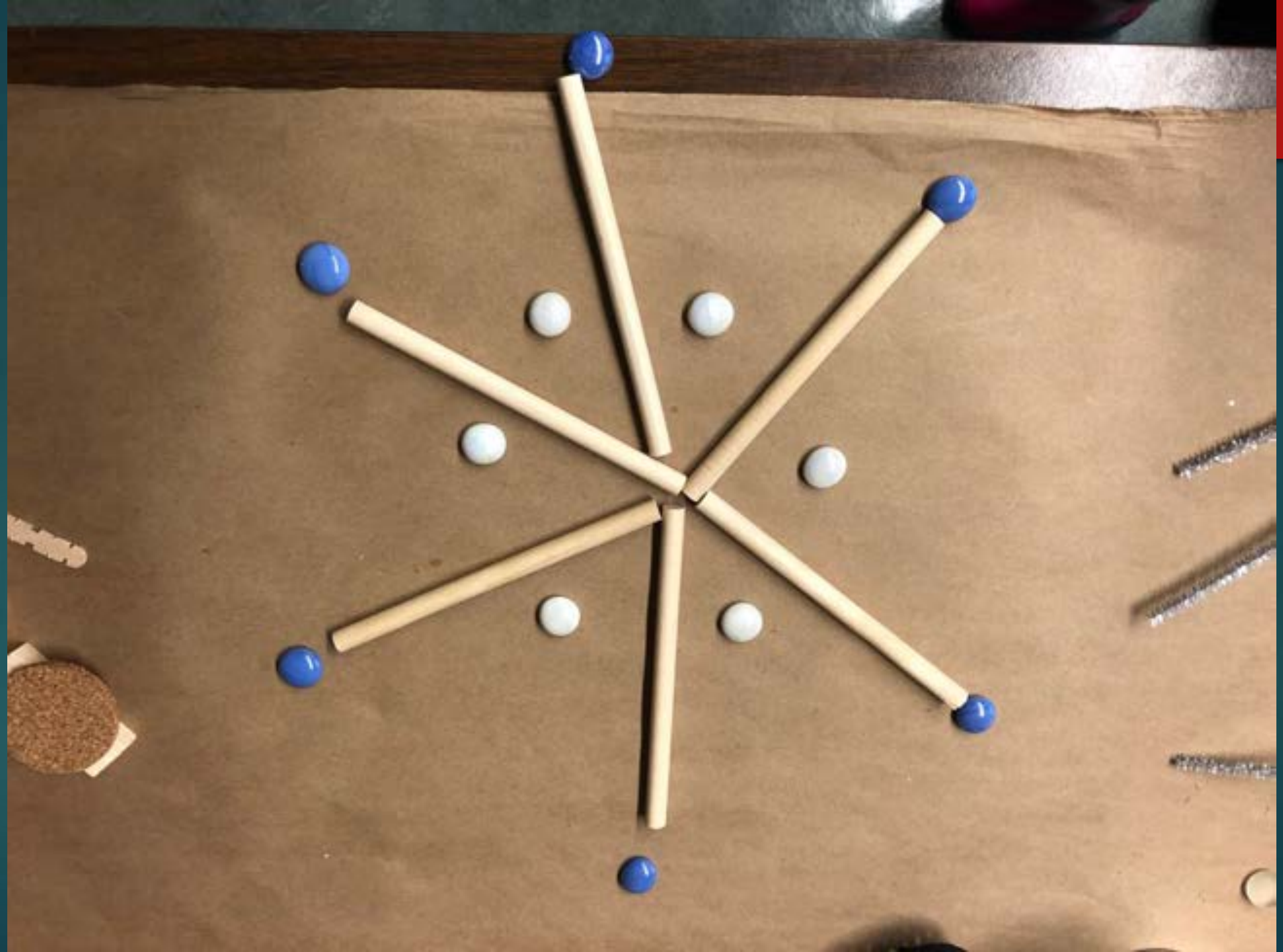


Investigating Snowflakes

- playing with ideas of size, shape and symmetry

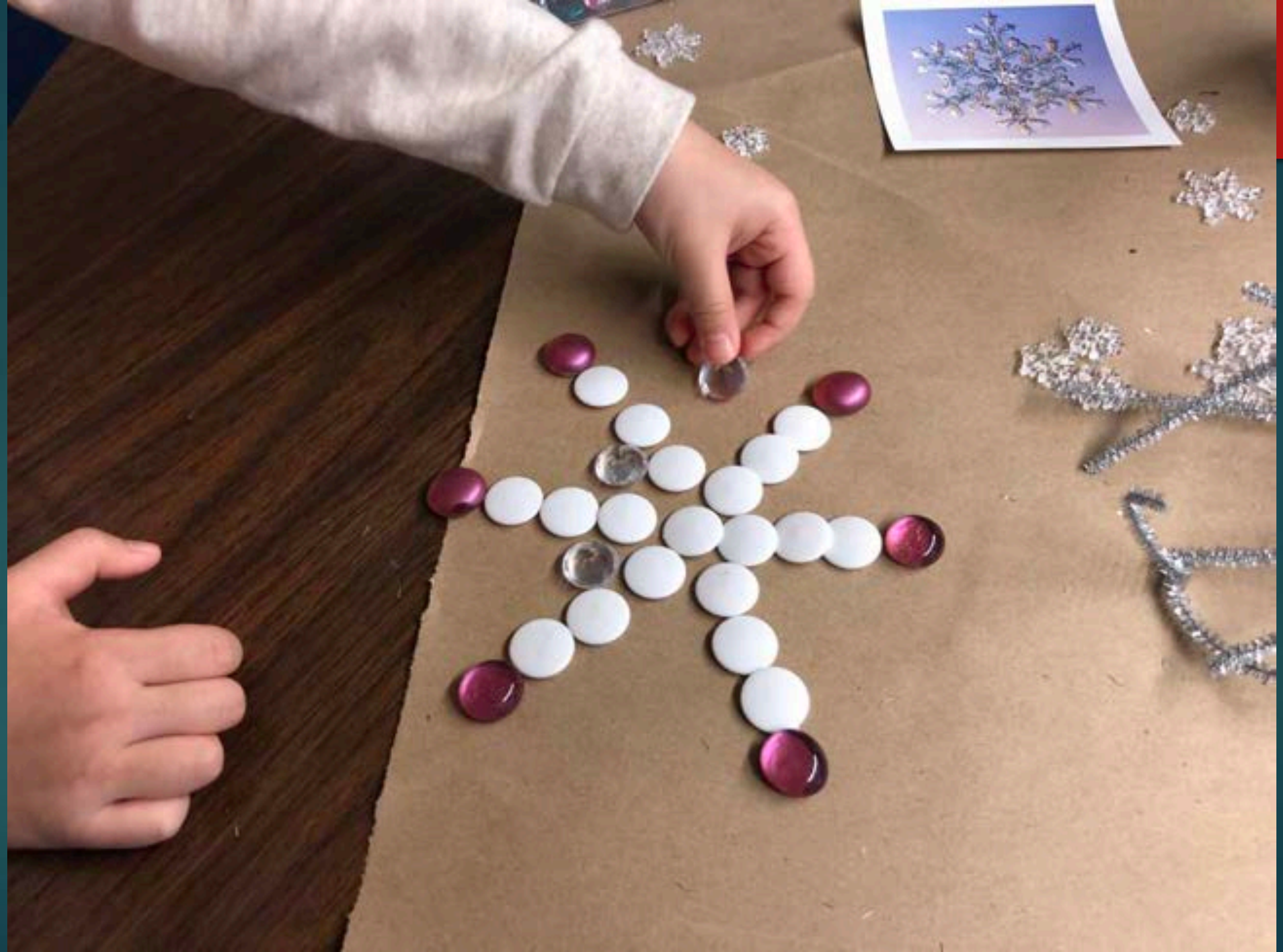

notice
• they are different
• the middles are not the same
• the lines go in different directions
6 sides
6 sides


What math
do you see?
• arrows
• lines
• circles
• hexagons
• shapes
• different sizes of shapes
• triangles
• six arms
• same on both sides
hexagon



Questions to support student inquiry around spatial reasoning:

- What stories live in these shapes?
- What is the relationship between 2D and 3D shapes?
- How can you combine shapes to make new shapes?
- What shapes live in this shape?
- How do shape and size help you make a map?
- How does visualizing help you make sense of the world?



Malke Rosenfeld
@mathinyourfeet


ten tiny things
TWO

Math to World Connections: Botany

- *connecting our thinking about size, shape and symmetry*


1
2
3
4
5
count the petals
• shapes
circle
star
lines
triangle
• sizes
• grow in groups
• hanging down
• different colours
Where do we see lines, shapes and

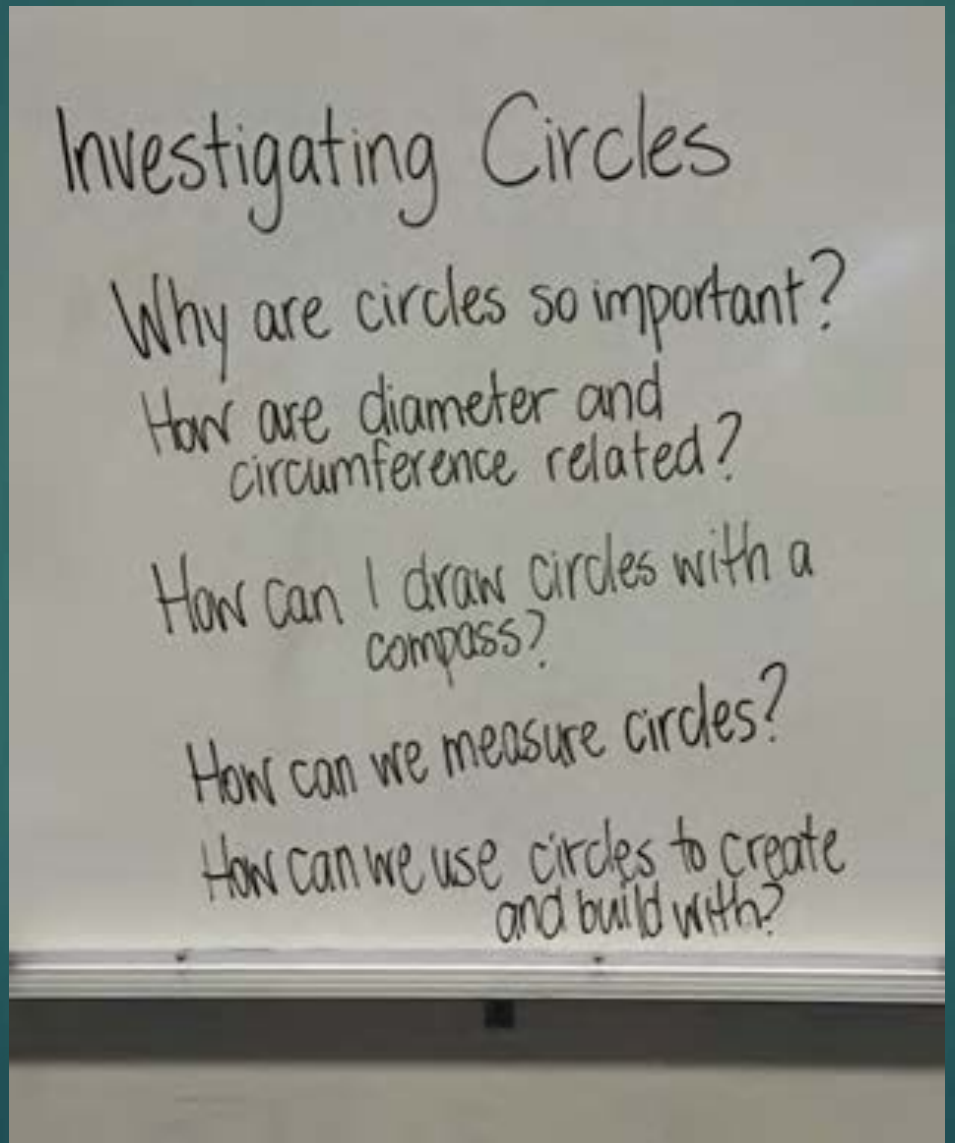

Acier inoxydable

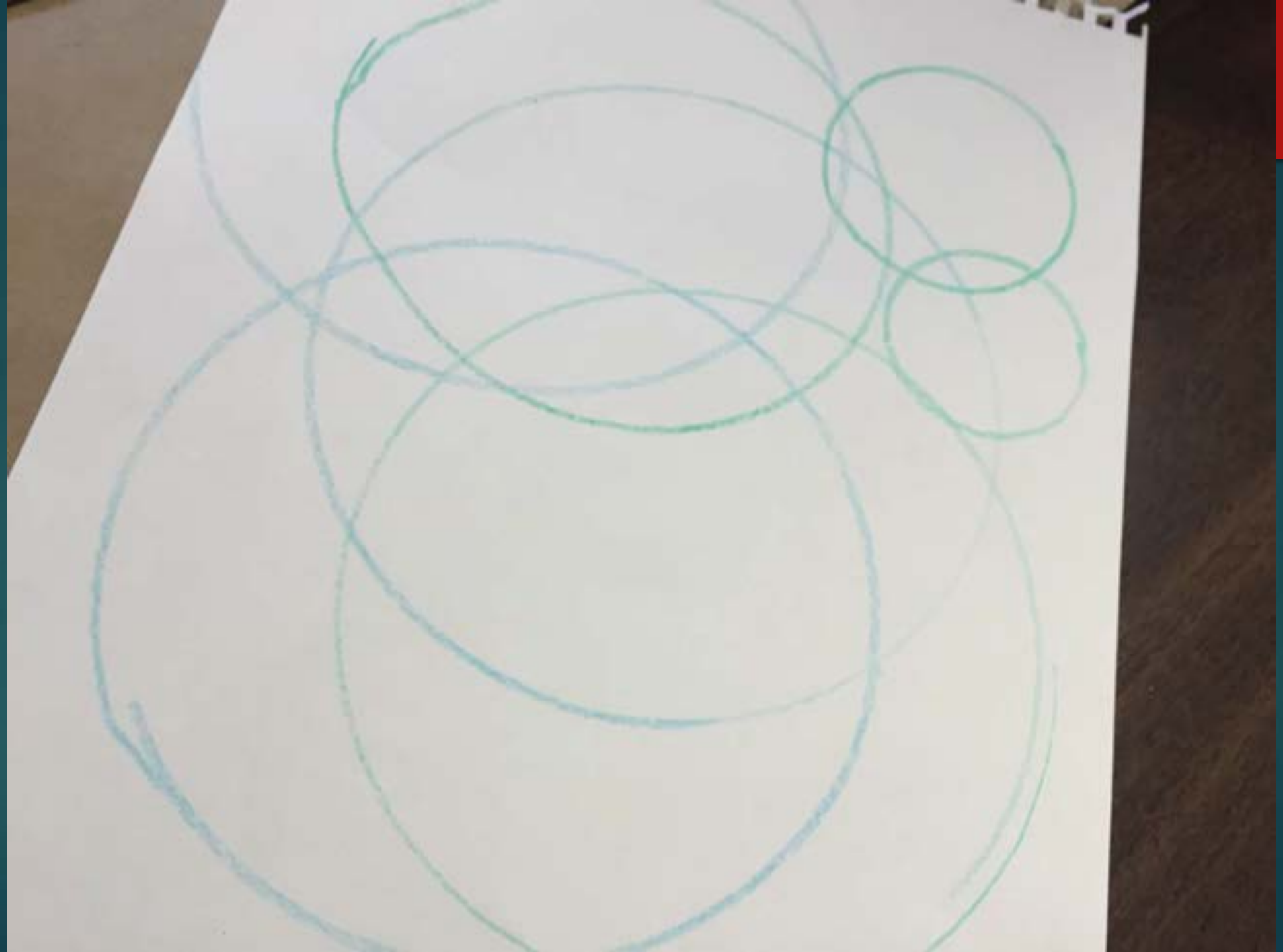

Black Spruce
Douglas-Fir
Oregon White Oak
Pacific Madrone
Hawthorn
Black Cottonwood

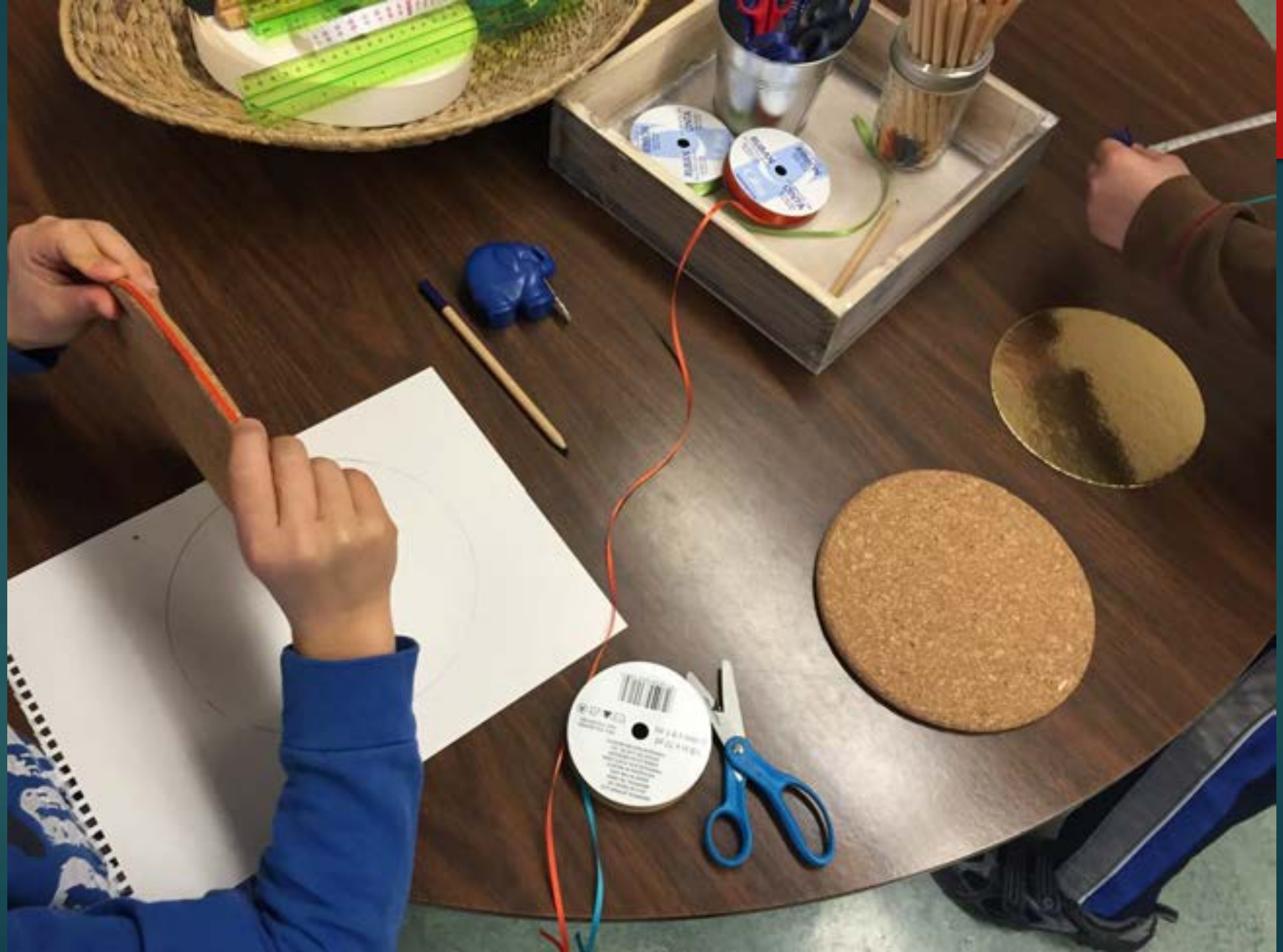
What connections are you making to your context?

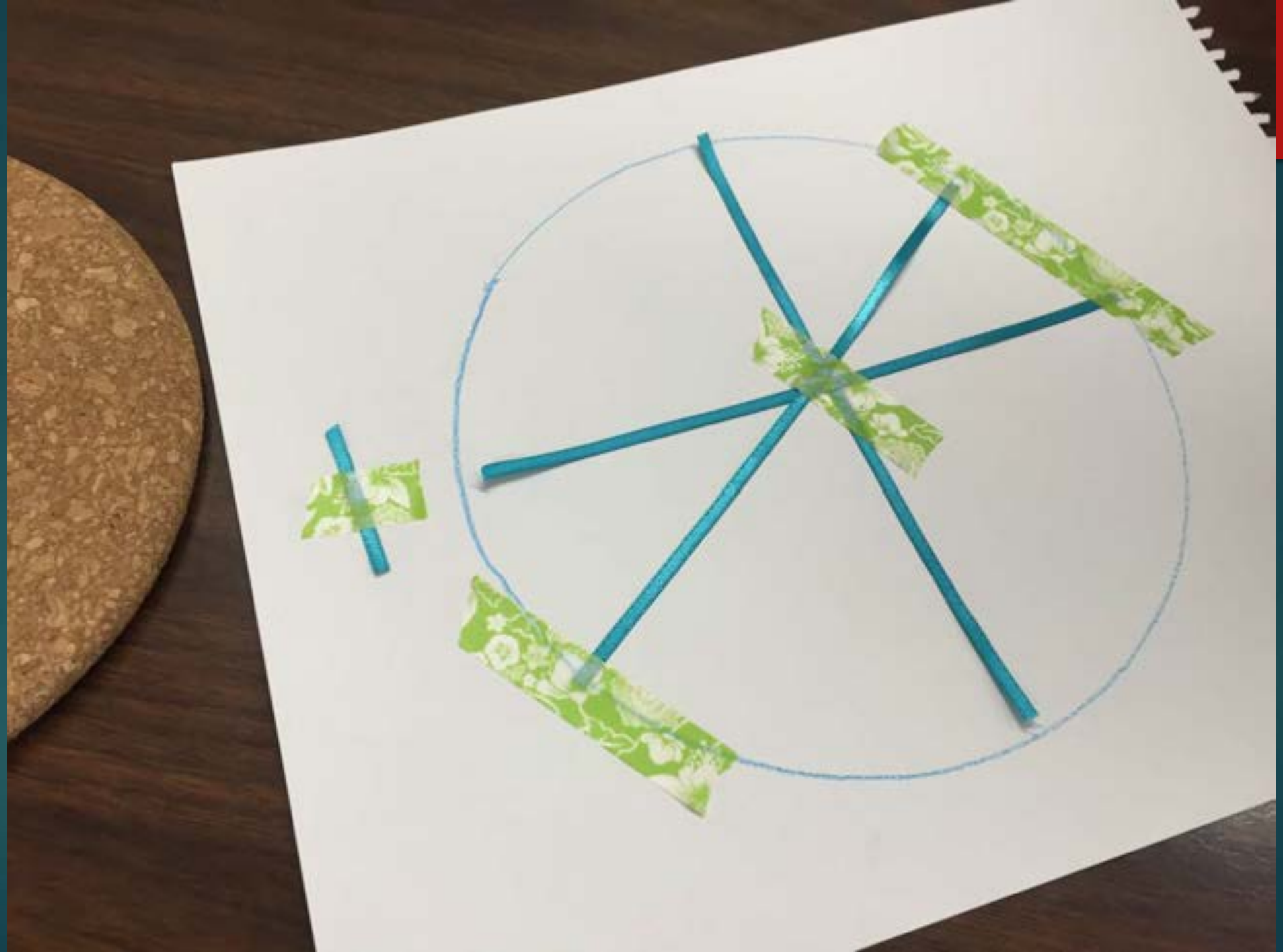
- Think of a mathematical big idea or concept and consider materials or investigations that might support connection-making for your students.
- What kind of studio-like experience could you provide for your students?

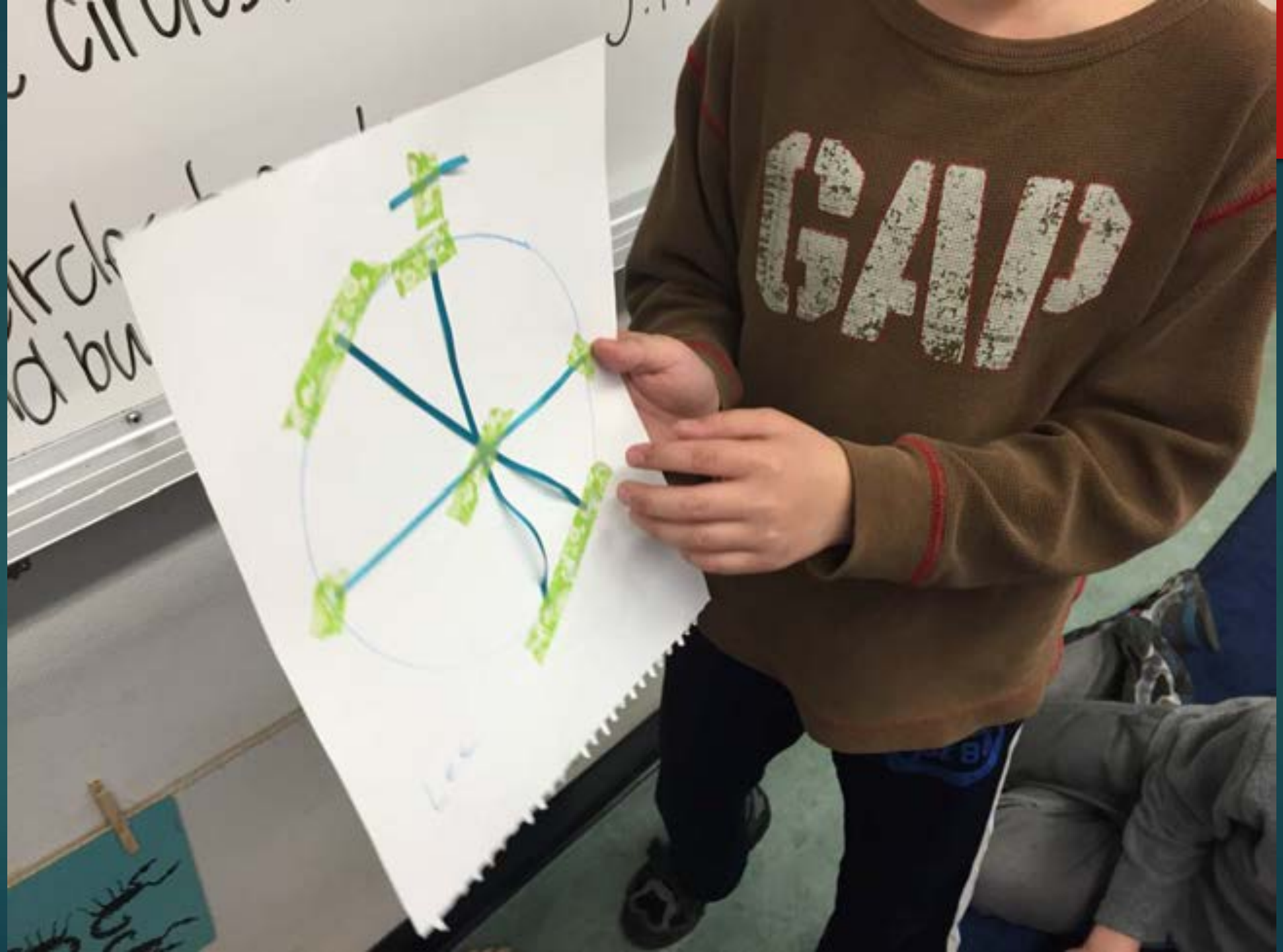
Investigating Circles

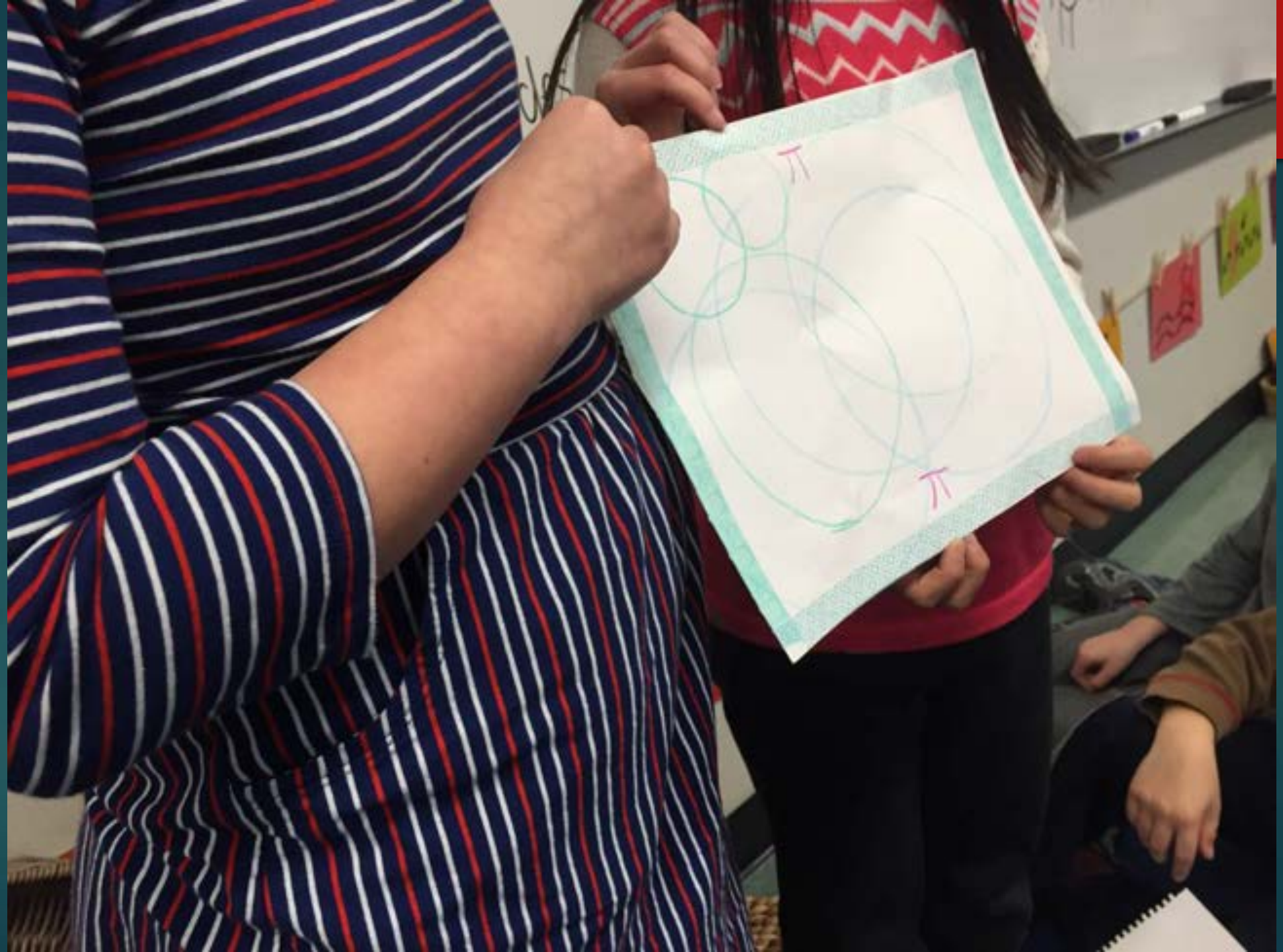






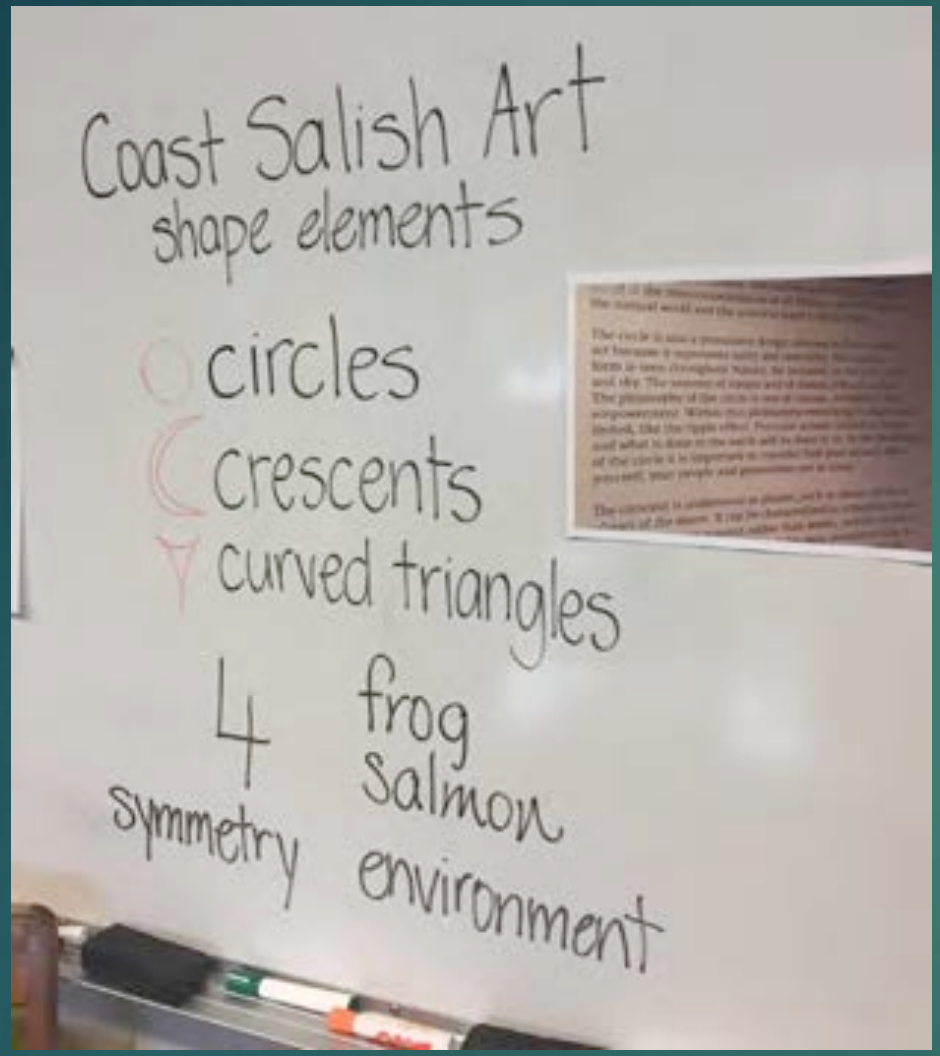



GAP



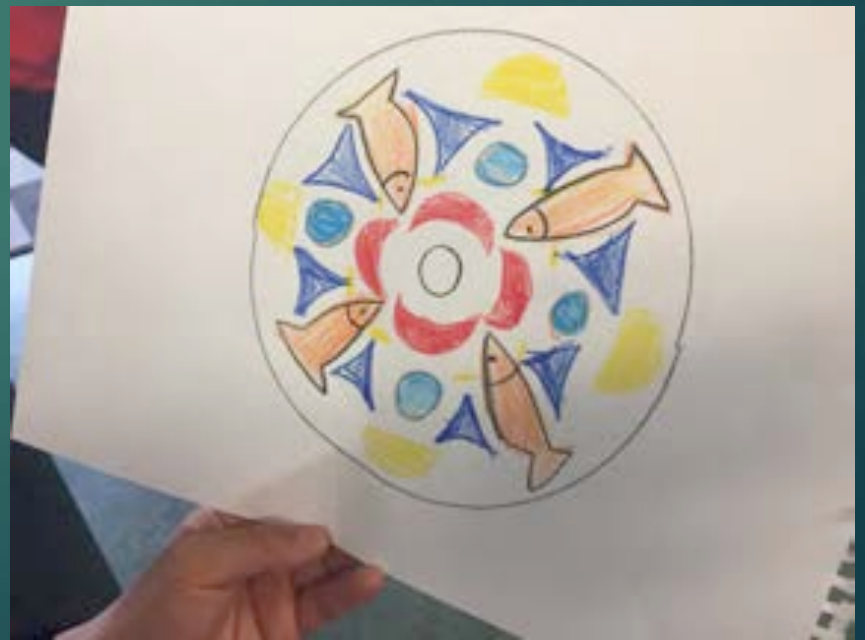
Thinking about the Circle:

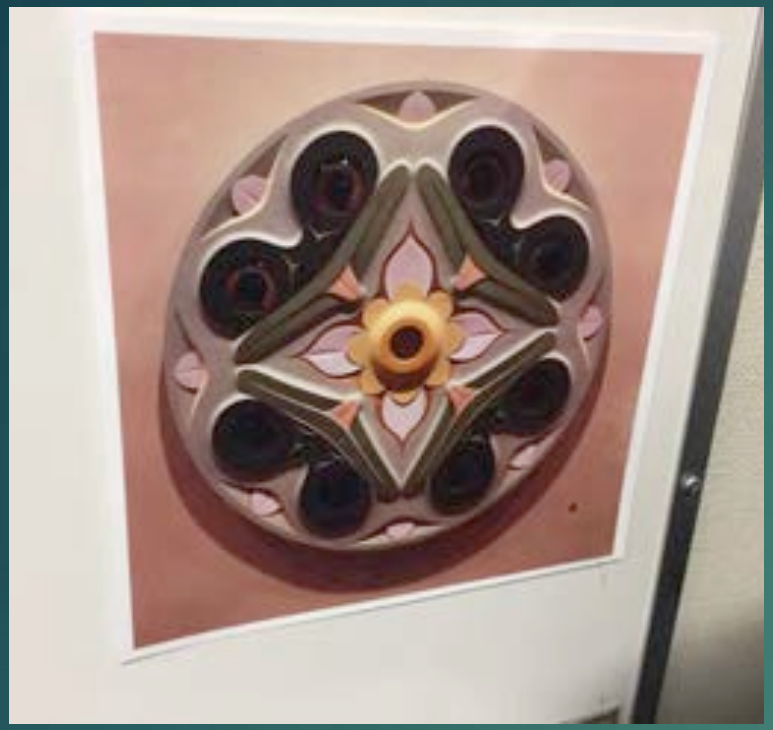
Where does math live in Susan Point's spindle whorl art?

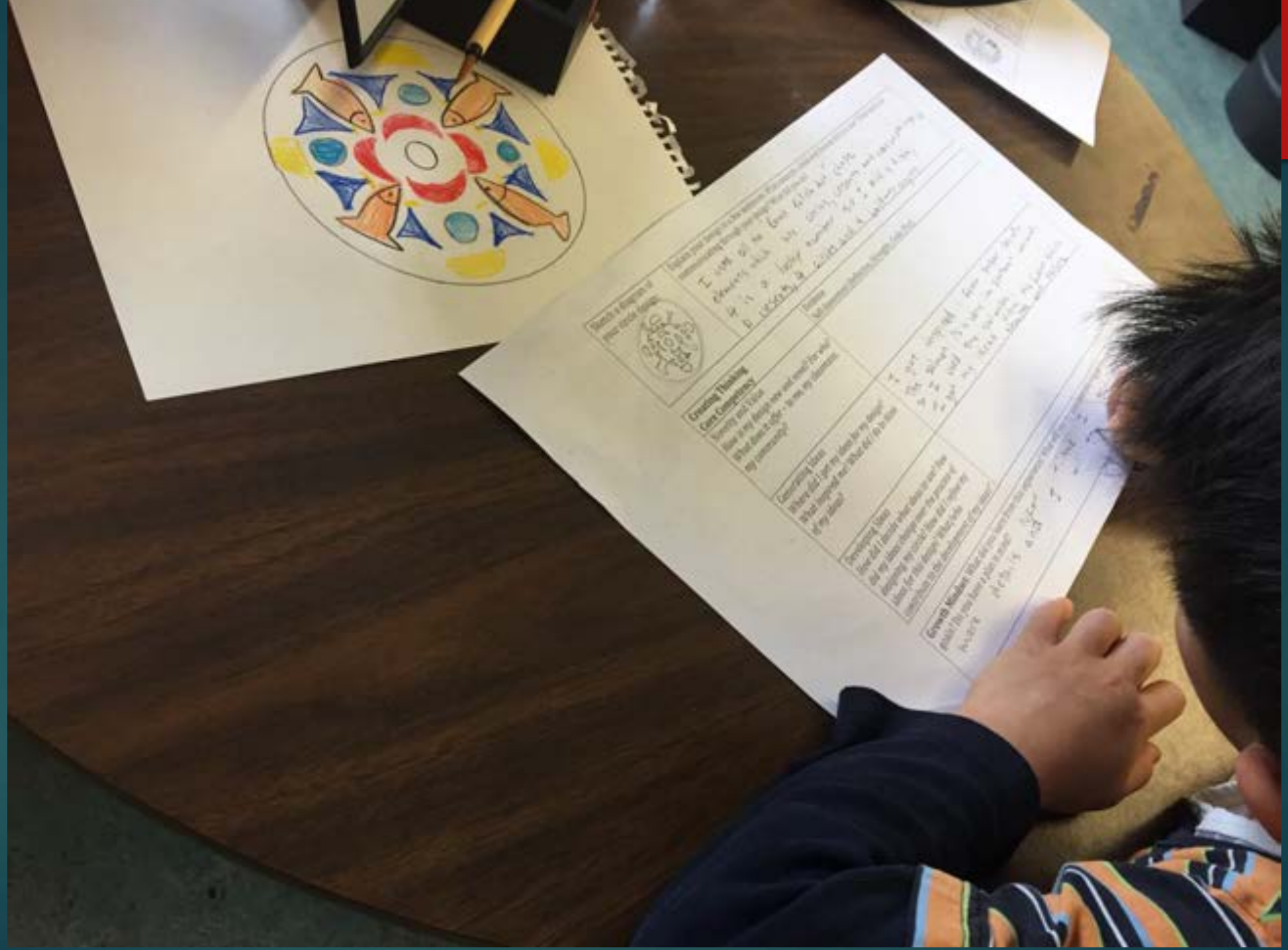

Coast Salish Art
shape elements
circles
crescents
curved triangles
4
frog
salmon
symmetry
environment

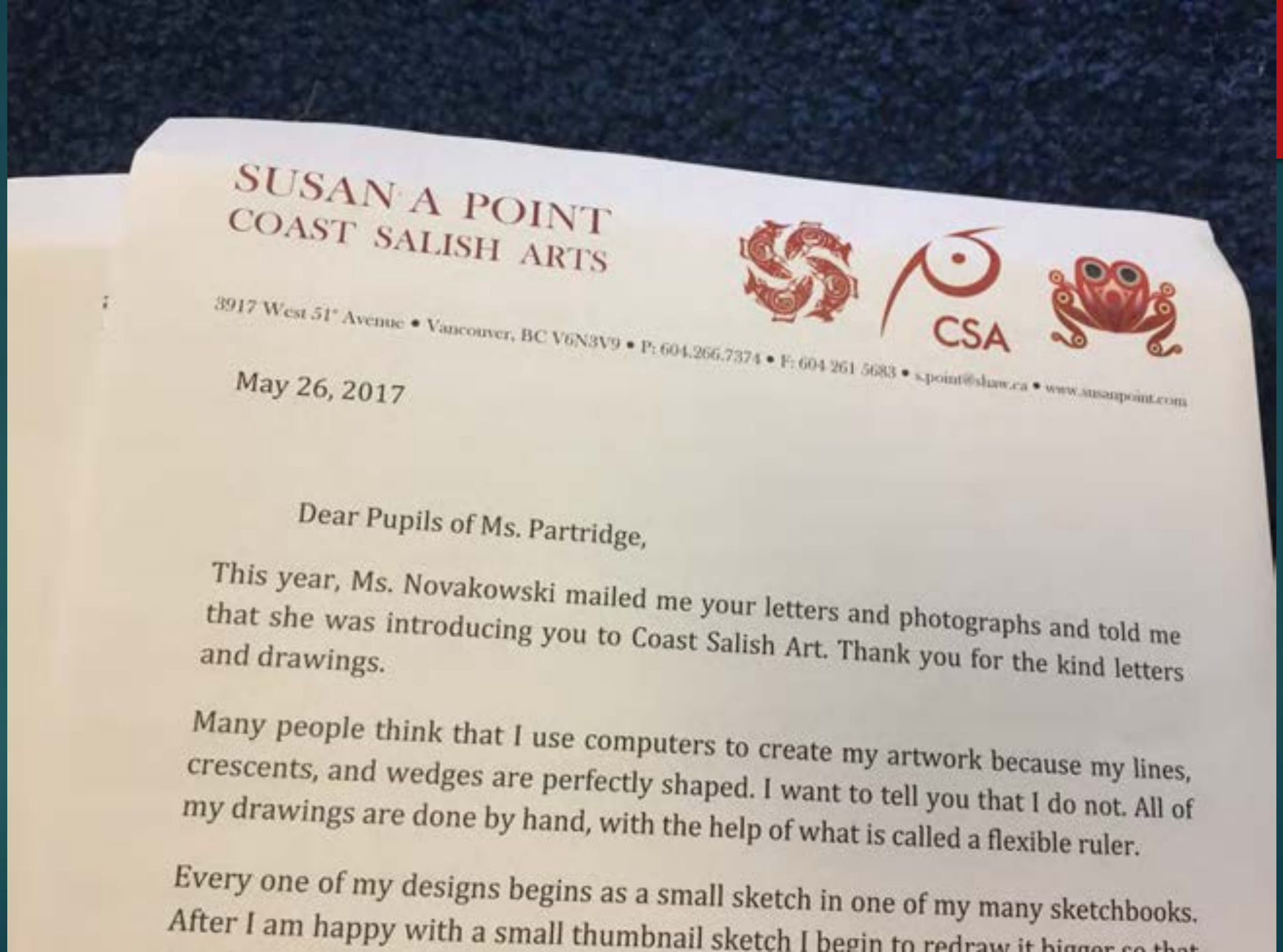

SUSAN
POINT
SPINDLE
WHORL


10









SUSAN A POINT
COAST SALISH ARTS

3917 West 51st Avenue • Vancouver, BC V6N3V9 • P: 604.266.7374 • F: 604 261 5683 • s.point@shaw.ca • www.susanpoint.com

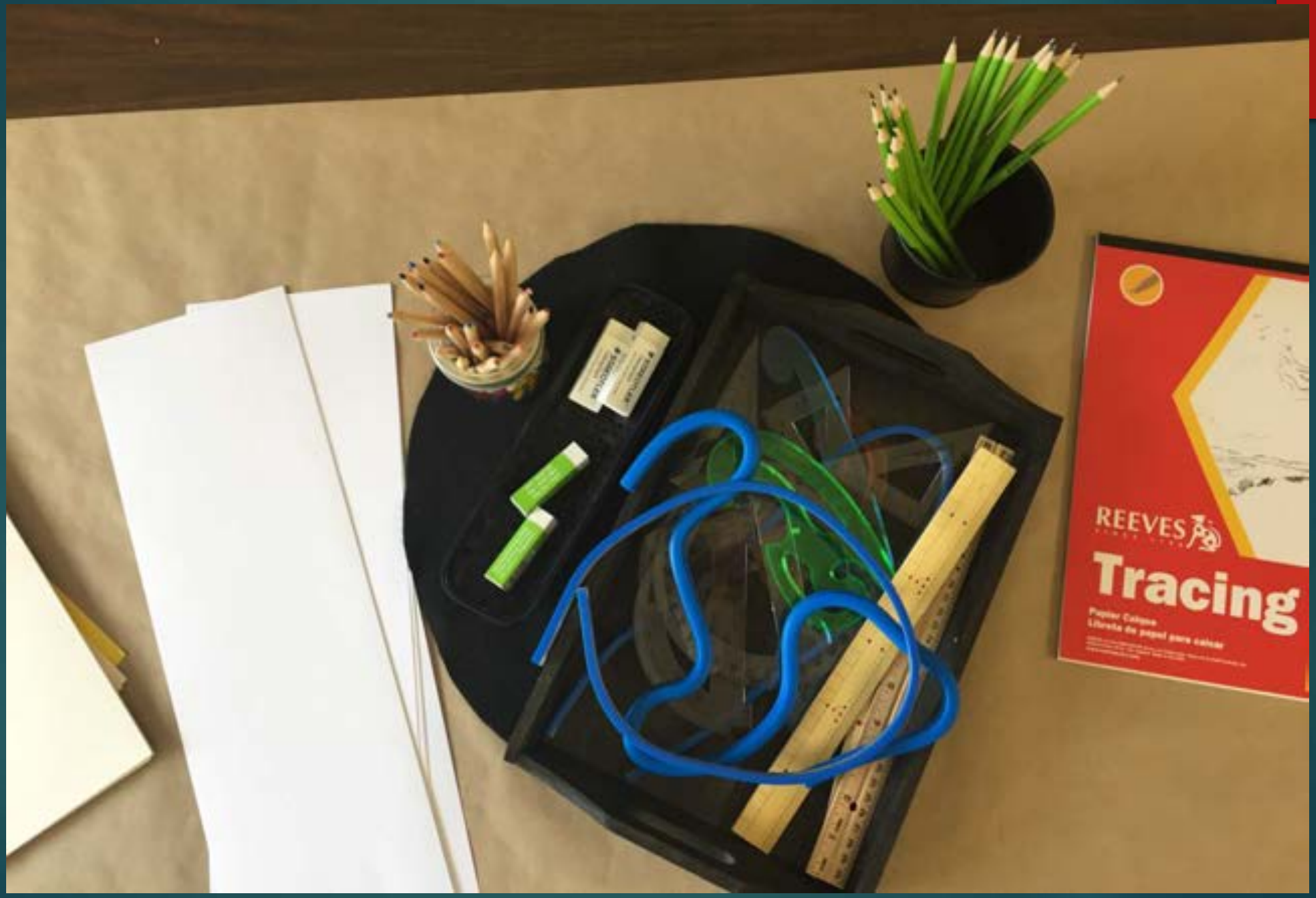
May 26, 2017

Dear Pupils of Ms. Partridge,

This year, Ms. Novakowski mailed me your letters and photographs and told me that she was introducing you to Coast Salish Art. Thank you for the kind letters and drawings.

Many people think that I use computers to create my artwork because my lines, crescents, and wedges are perfectly shaped. I want to tell you that I do not. All of my drawings are done by hand, with the help of what is called a flexible ruler.

Every one of my designs begins as a small sketch in one of my many sketchbooks. After I am happy with a small thumbnail sketch I begin to redraw it bigger so that

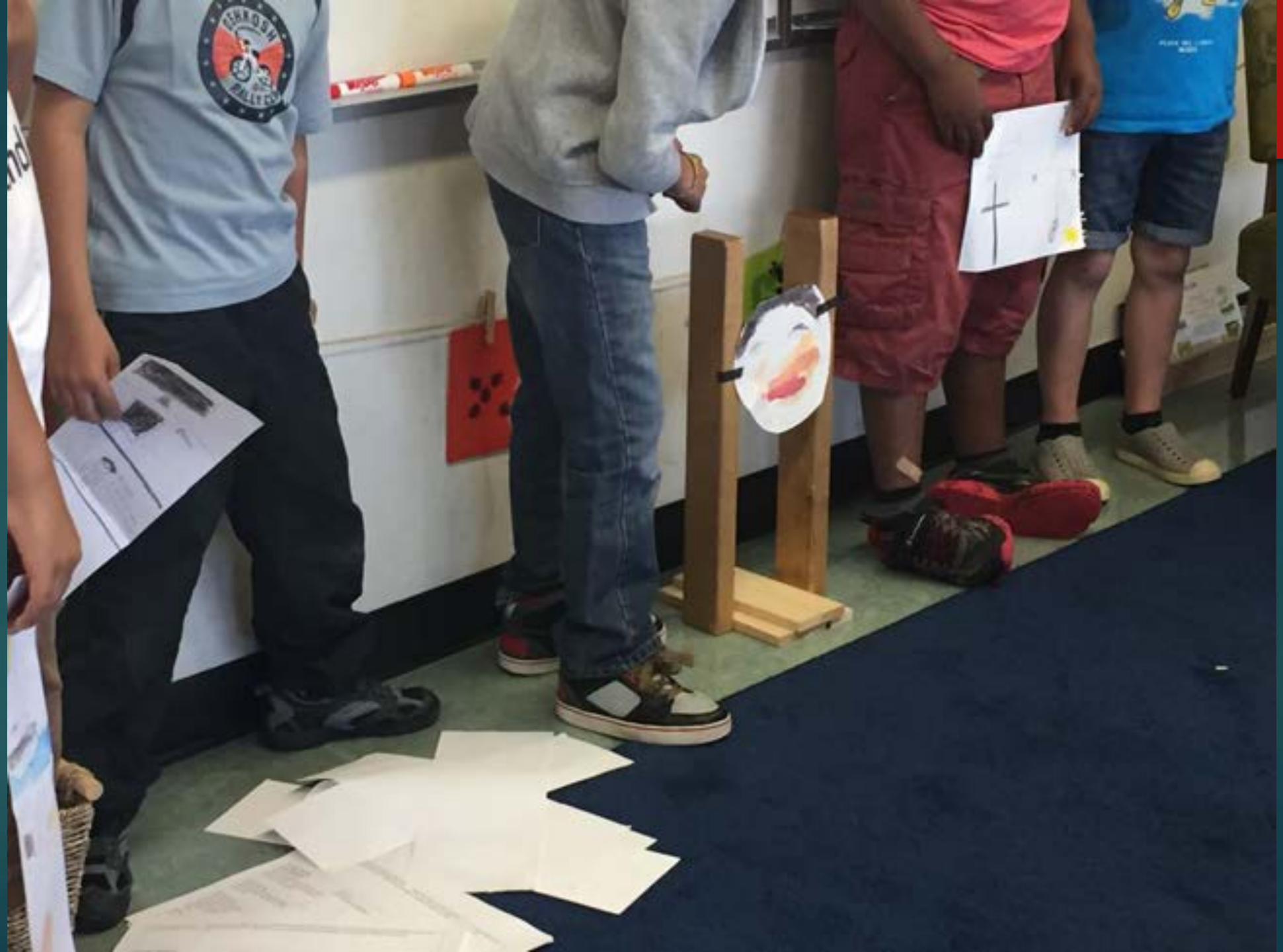

REEVES
Tracing

MUSQUEAM WELCOME

Susan Point


Hiking
Mountain
Hockey
Pizza

ਪਰਿਵਾਰ



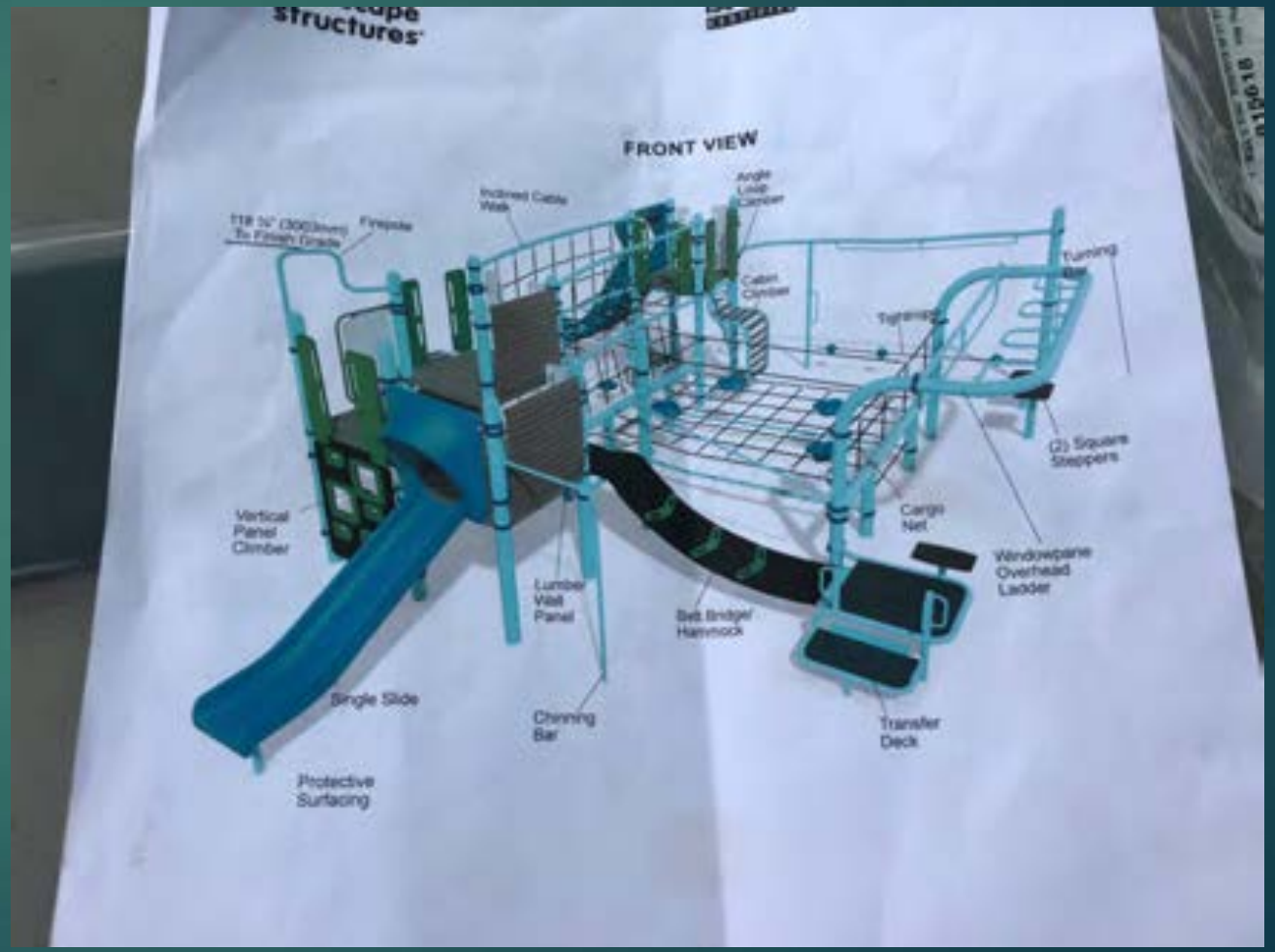

10

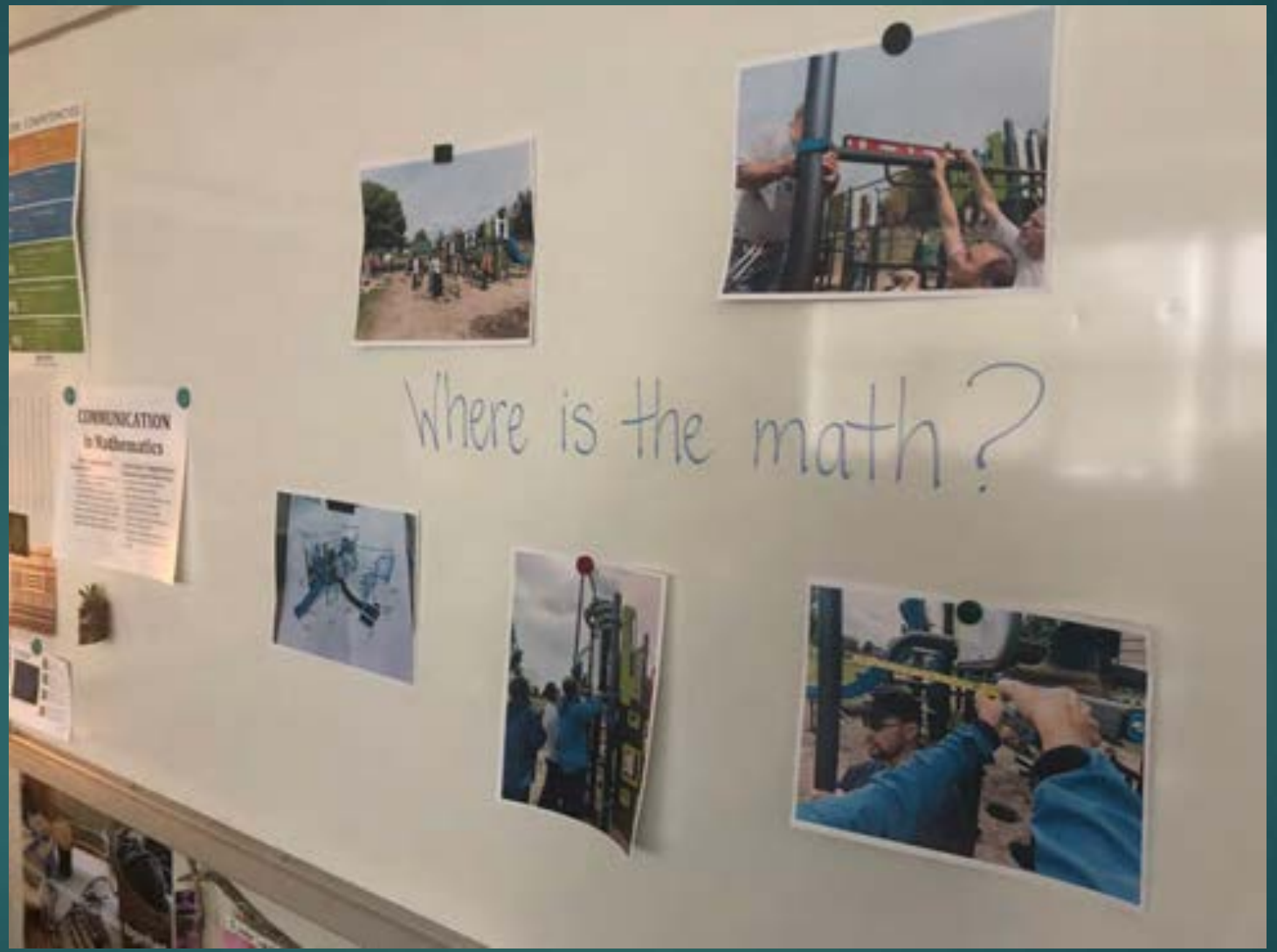
What local artists or pieces of art might inspire mathematical thinking?

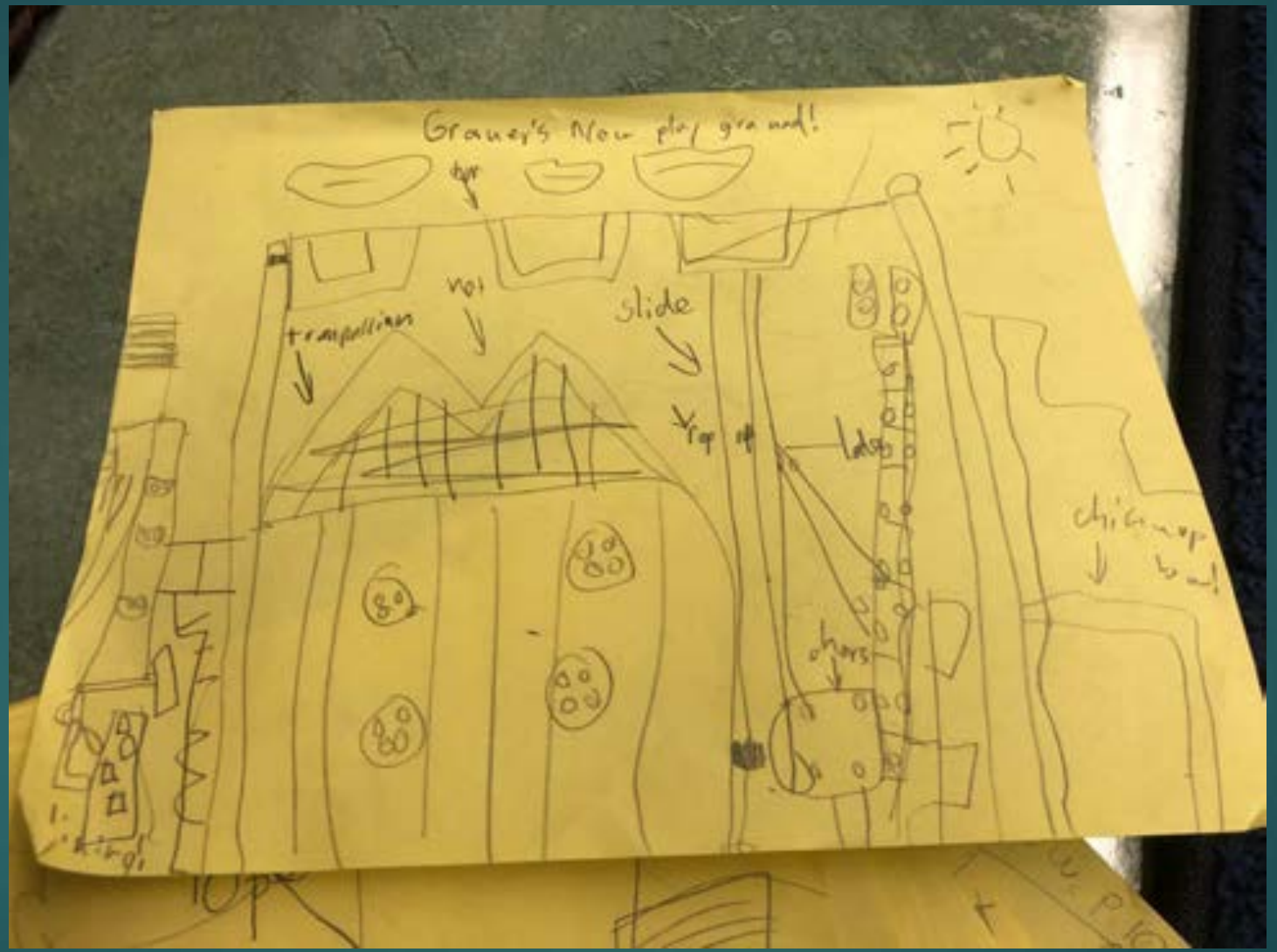
The Playground Project

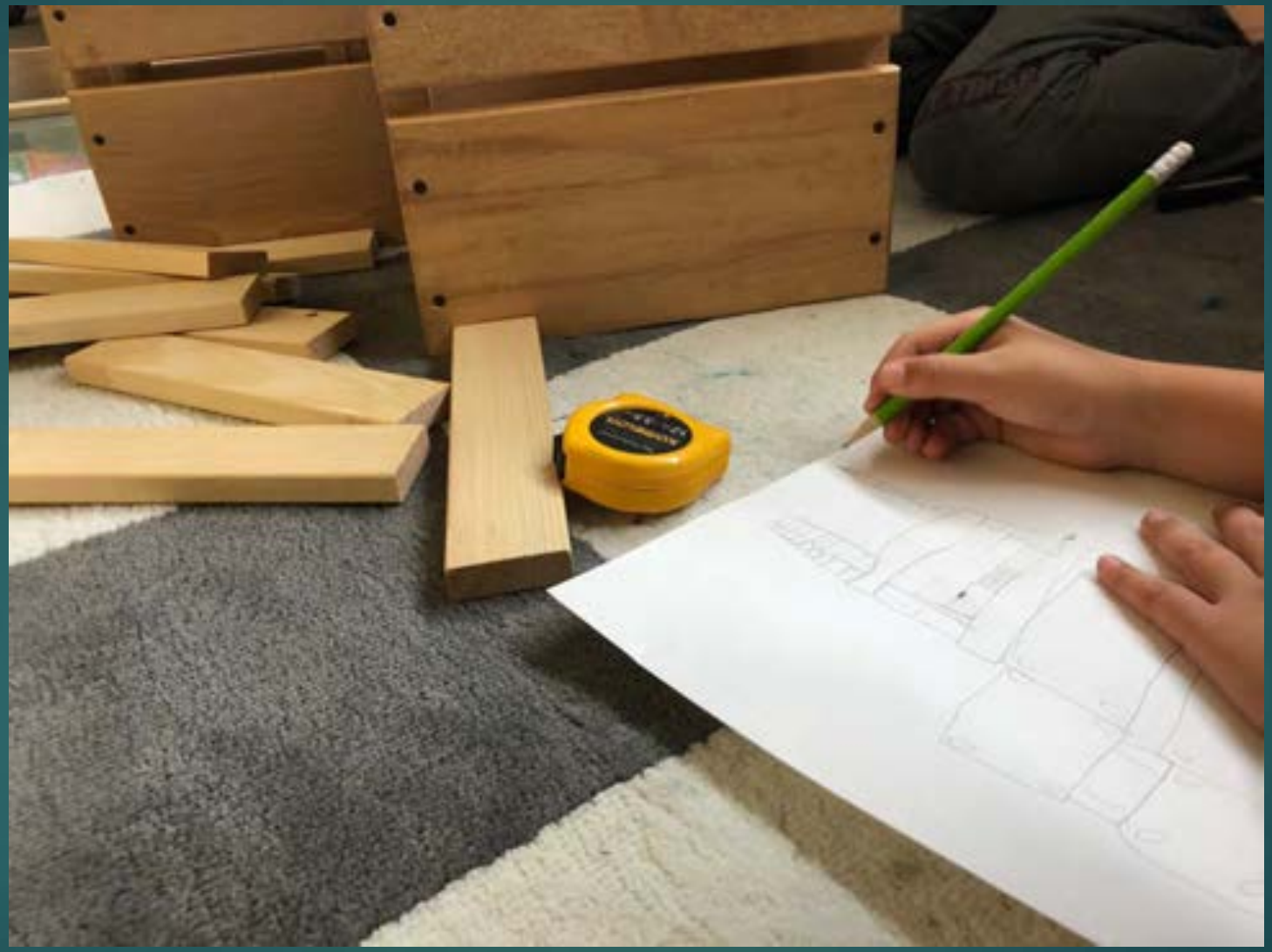
-playing with ideas of
shape and size
and spatial reasoning

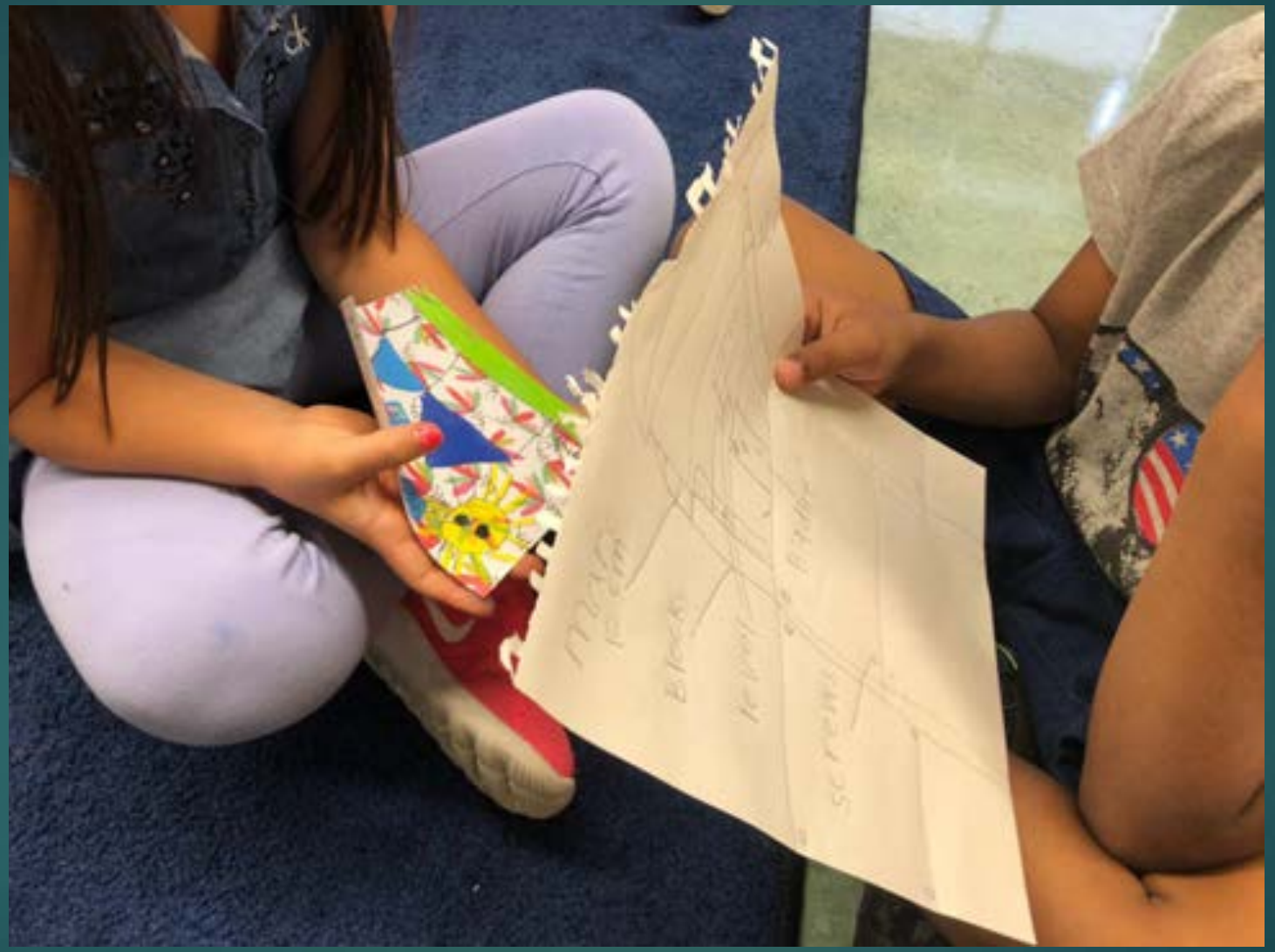

Help us build our dream playground!
Stay tuned for Phase 2
GRAUER
$60,000
$50,000
$40,000
$30,000
$20,000
$10,000

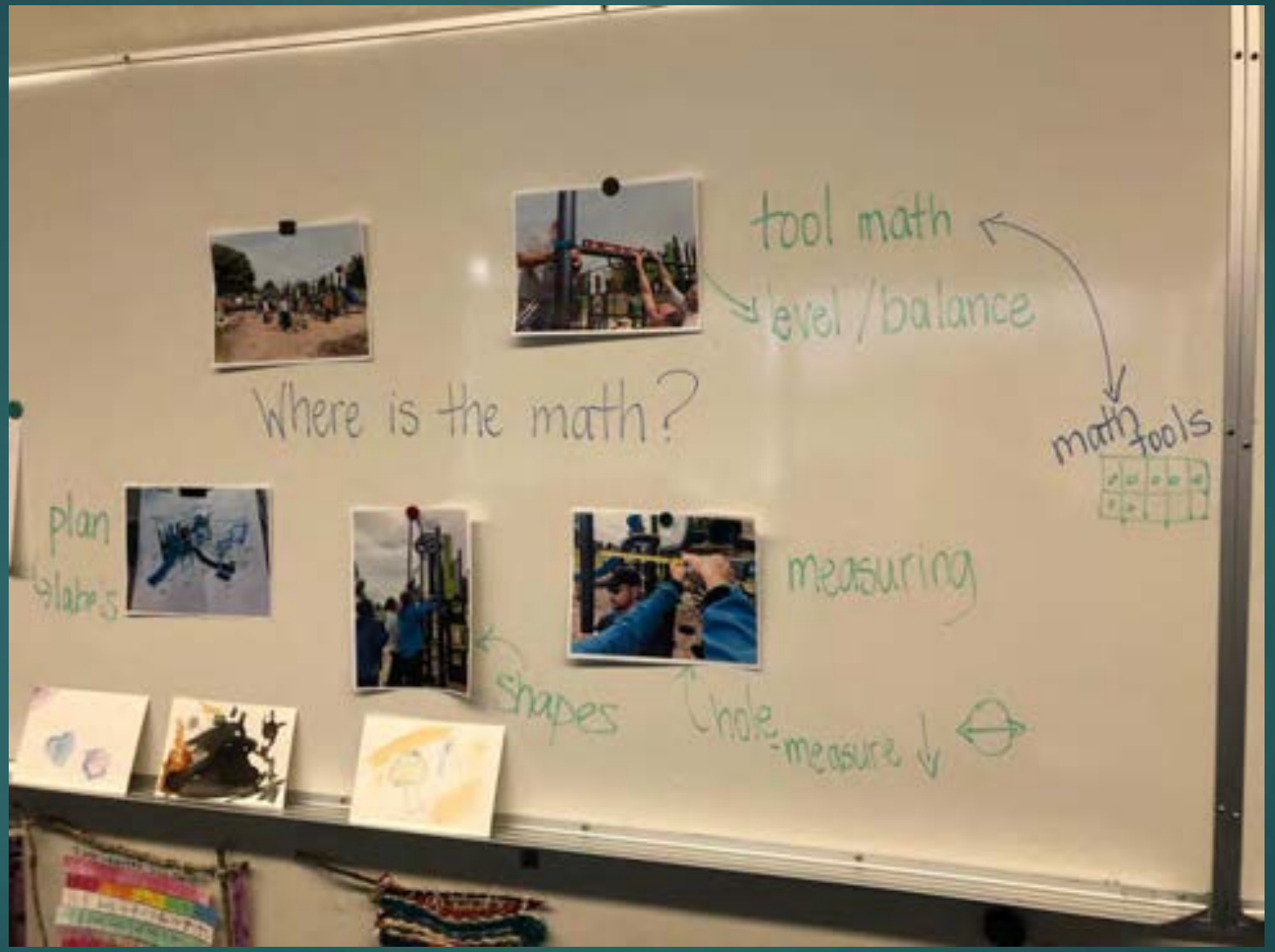

FRONT VIEW
Fireplate
Inclined Cable Walk
Angle Loop Climber
Cabin Climber
Tightrope
Turning Bar
(2) Square Steppers
Vertical Panel Climber
Cargo Net
Windowpane Overhead Ladder
Lumber Wall Panel
Belt Bridge/ Hammock
Single Slide
Chinning Bar
Transfer Deck
Protective Surfacing


COMMUNICATION
in Mathematics
Where is the math?

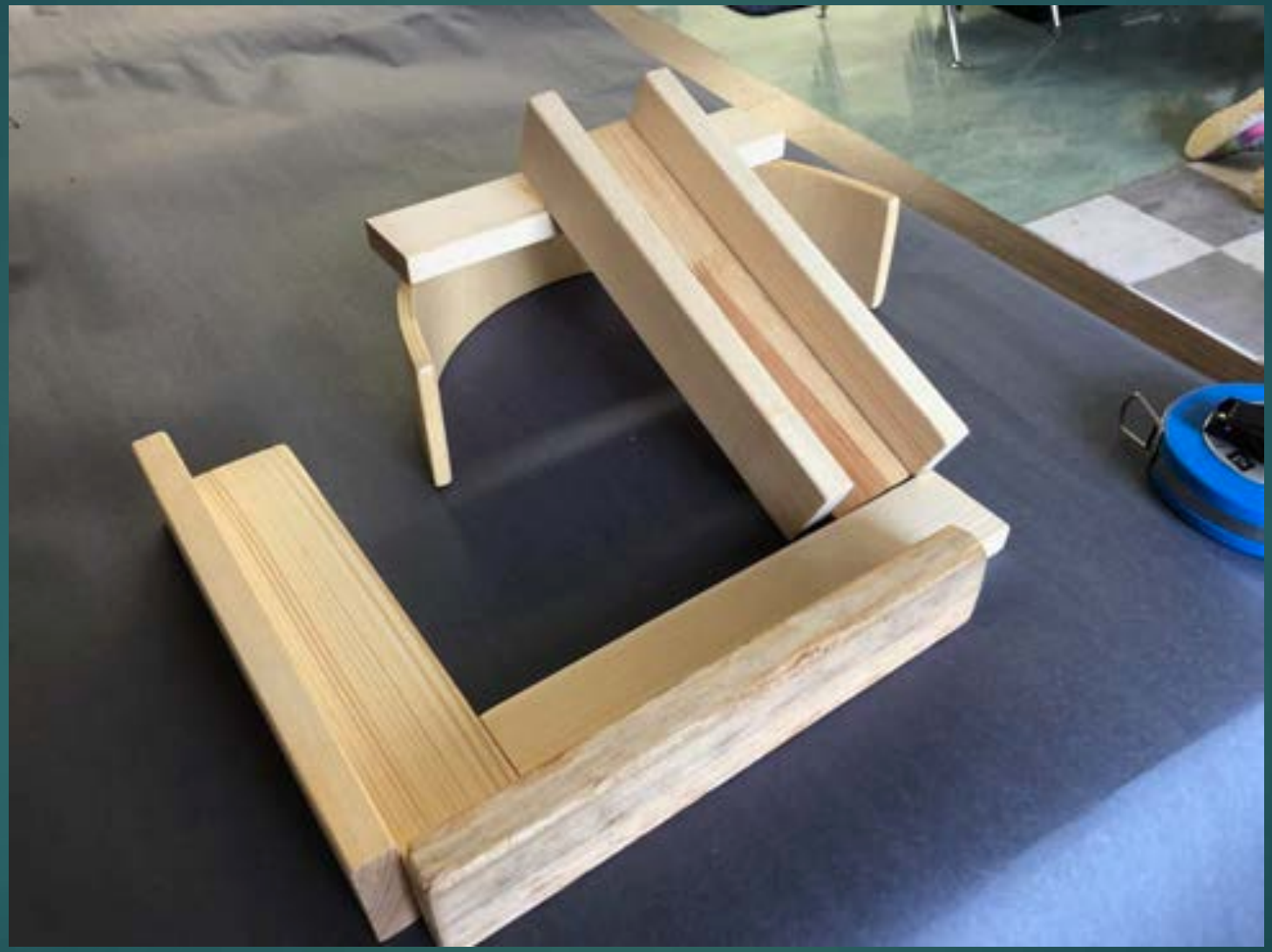

Grauer's New play ground!
trampolines
rail
slide
chairs

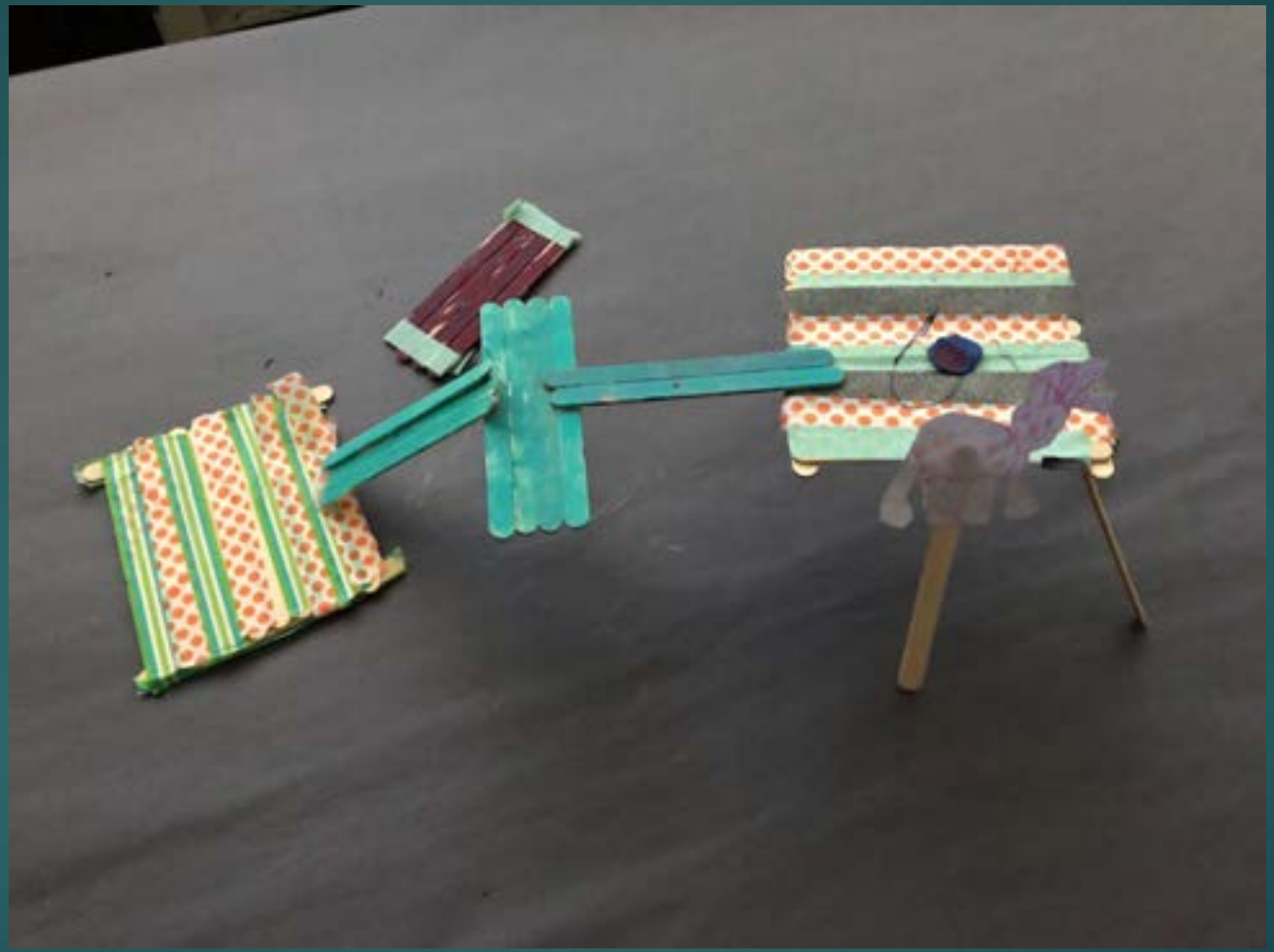





tool math
level / balance
Where is the math?
math tools
plan
measuring
shapes
whole-measure


Toy Stories
Let's Build a Playground





following a plan, reading diagrams, labels and specifications

WHERE IS THE MATH?

Divisions 5 & 6 (grades 1 and 2) considered where, when and how mathematics was used to install Grauer's new playground. We first had to think about "what is math?" and broaden our thinking beyond thinking math is only about numbers. Students began to connect the playground installation to other authentic experiences they have had in which they used mathematics

measuring distance

using tools level and balance

measuring depth

shapes and design

Where, when and how do we use mathematics in our lives?

September 2018

"a playground for animals"

DESIGNING PLAYGROUNDS

Divisions 5 & 6 (grades 1 and 2) have been inspired by the installation of the new playground at Grauer. They were invited to design and create their own playgrounds and consider the mathematics embedded in the process of design and creation. The students were introduced to the idea of creating math maps to consider *where is the math? in* their playgrounds.

"We keep checking to see if its level so it won't fall down."

math maps

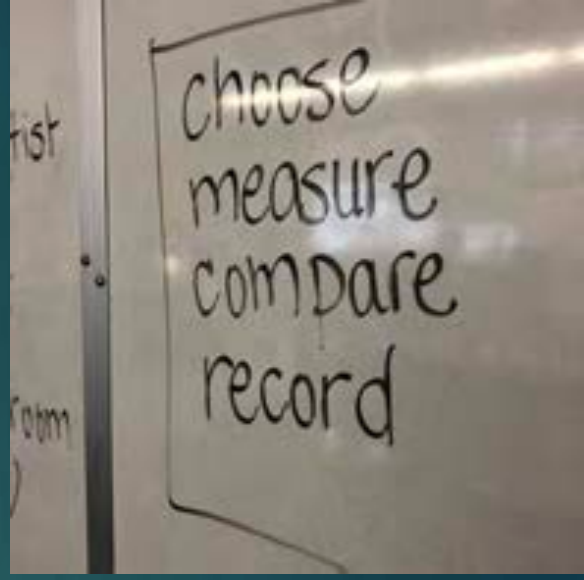
What mathematics do we use when designing and creating?

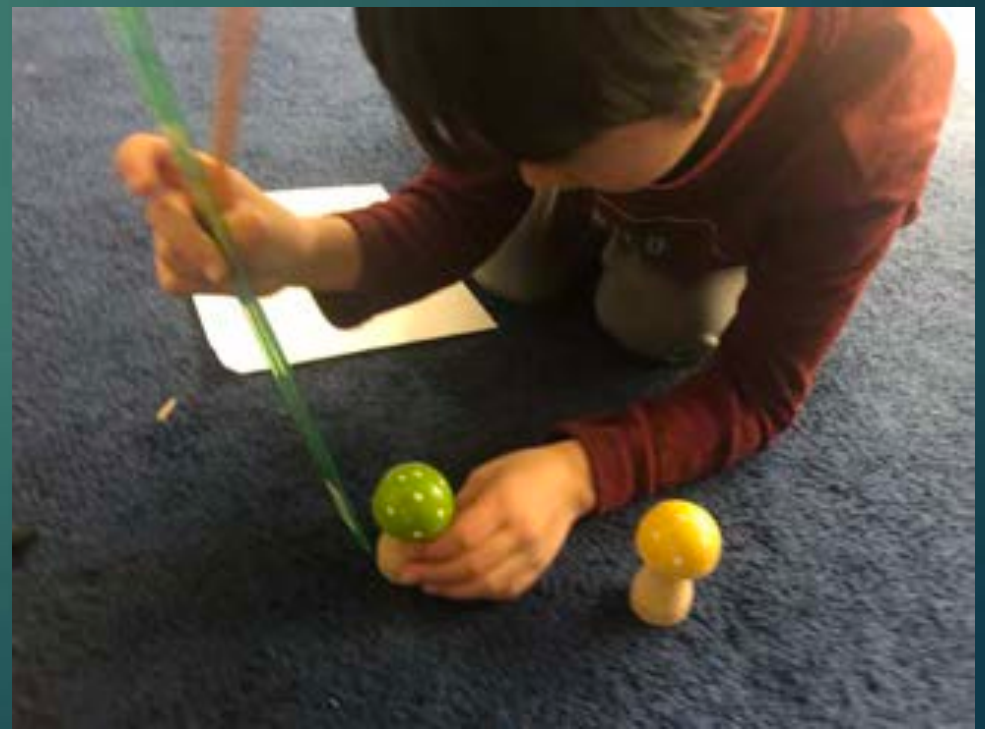
September 2018

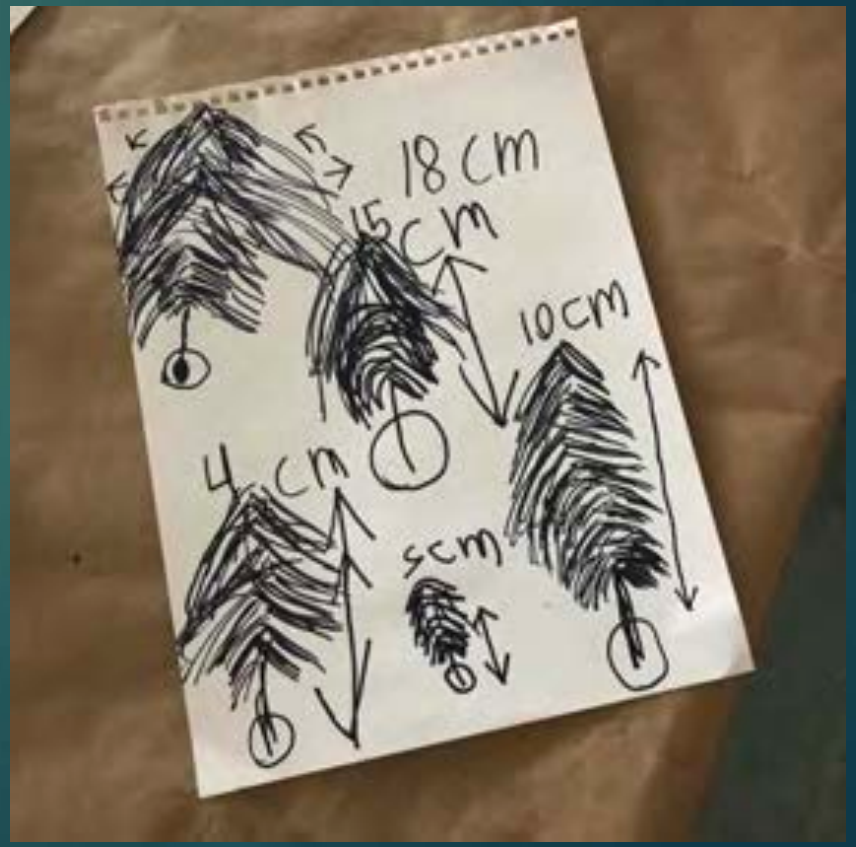
encounters with measurement

- What opportunities for measurement are there in your social studies, science, ADST, Arts or Physical Education curriculum?
  - In exploratory play? Outdoors?
- What tools or materials could be used to support and enhance students' understanding of measurement?

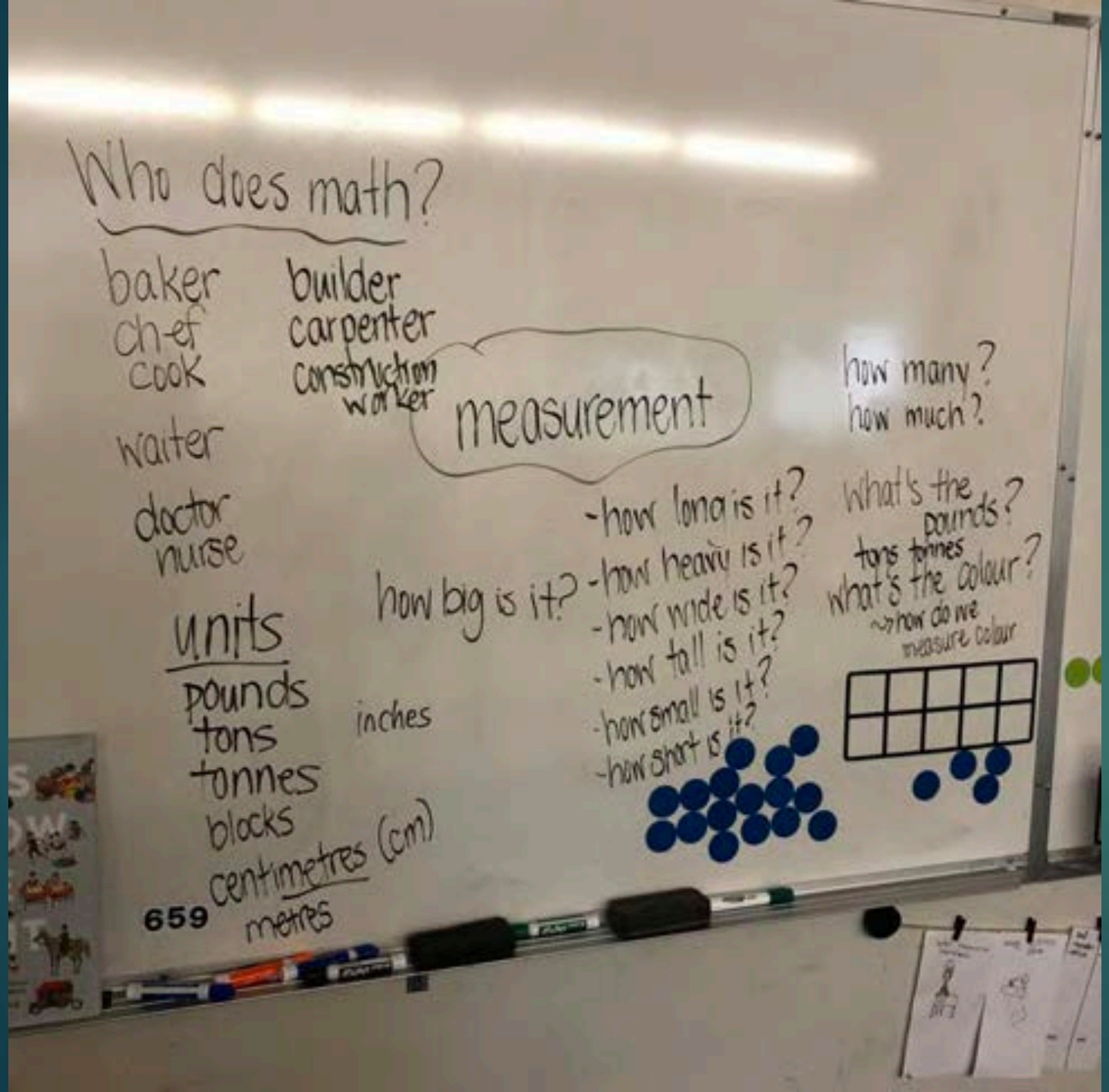
playing with the idea of measuring


choose
measure
compare
record

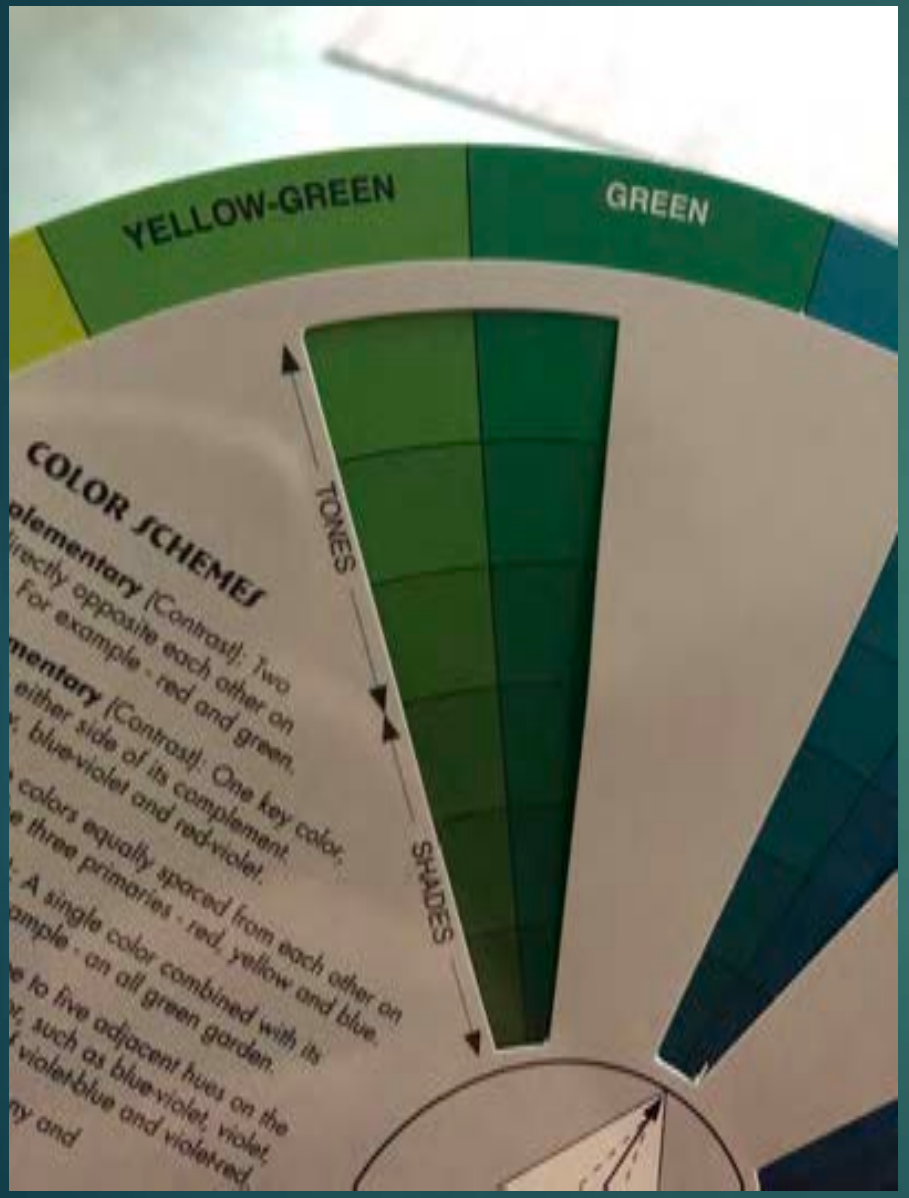



18 cm
15 cm
10cm
4 cm
5cm

What does it mean to measure?
What do we do when we measure?
How do we communicate measurements?
When do we measure in our daily lives?



In discussing attributes of objects that we can measure, a grade one student wondered, *How do we measure colour?*


YELLOW-GREEN
GREEN
TONES
SHADES
COLOR SCHEMES

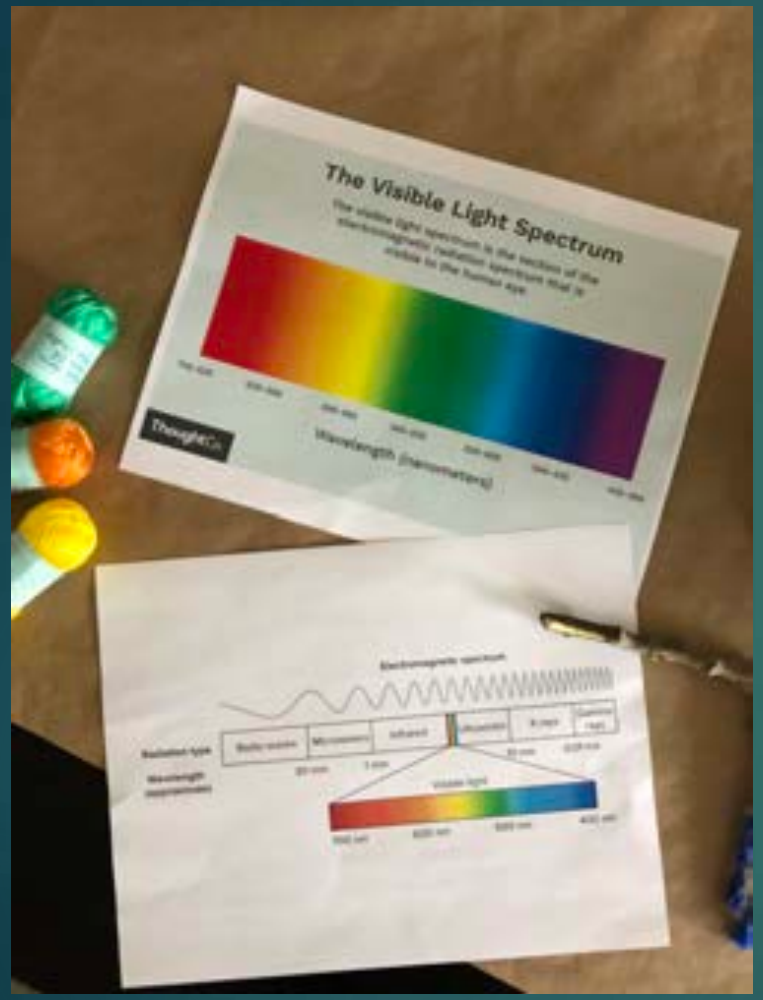

PANTONE 635
PANTONE 9044

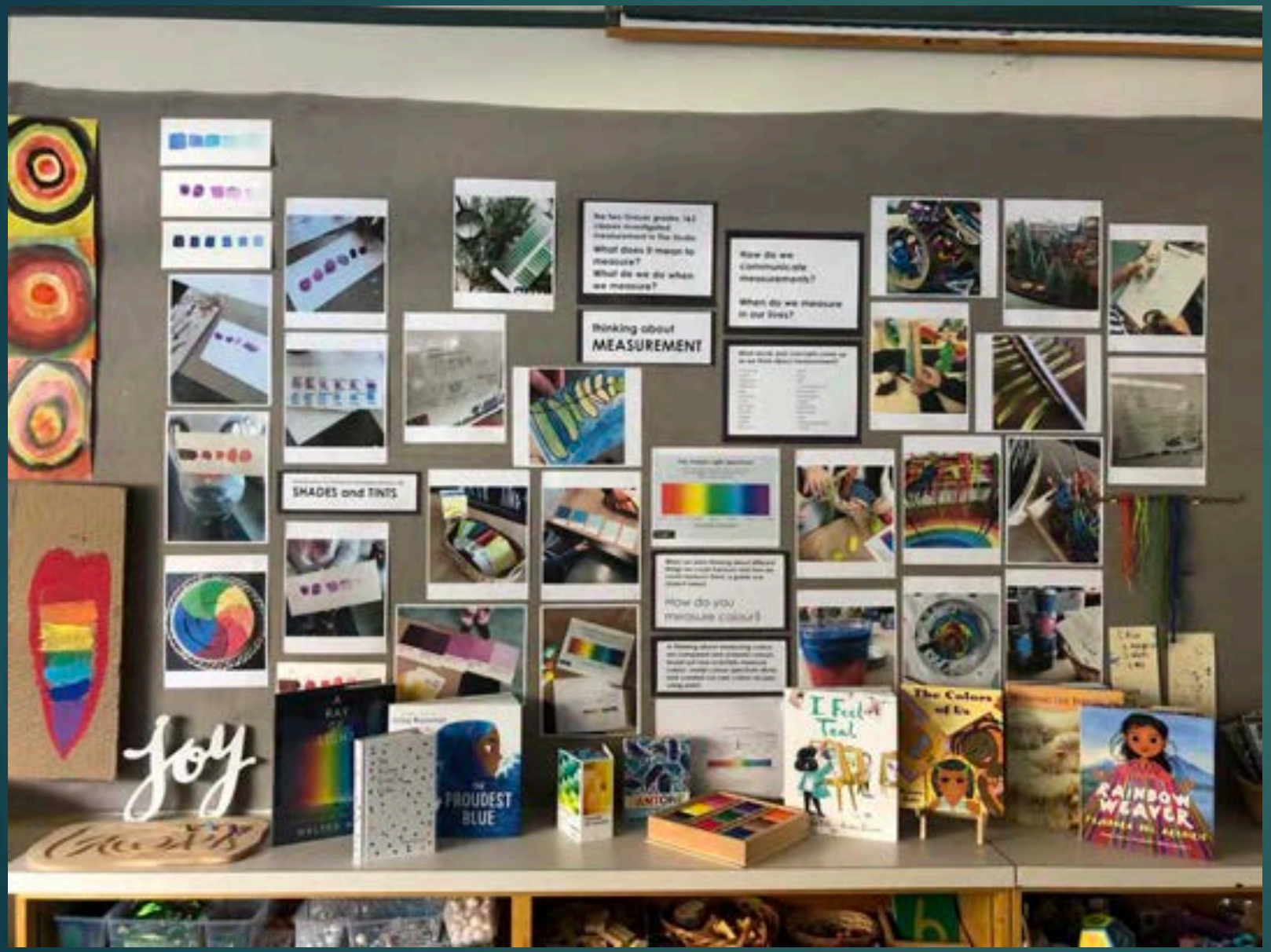

PANTONE 5125
PANTONE 19-3438
Bright Violet
PANTONE 18-3025
Striking Purple
PANTONE 7654
PANTONE 14-3612
Orchid Bloom
PANTONE 7548


Rose purple
Pink Daisy
Cotton Candy
Purple Grapes
Blueberry Pie
Blackberry Purple


create
LET'S PAINT!
I Feel Teal


I Feel Teal


The Visible Light Spectrum
Wavelength (nanometers)
Electromagnetic spectrum


thinking about
MEASUREMENT
SHADES and TINTS
How do you measure color?
joy
PROUDEST BLUE
I Feel Teal
The Colors of Us
RAINBOW WEAVER


The Colors of Us
Karen Katz

What is an interdisciplinary investigation that would connect and deepen understanding of a mathematical idea?

taking The Studio experience out in the community

- SD38 Math Play Space

- more info
  - NCTM Regional Boston 2019
  - https://bit.ly/JN_NCTM_Boston

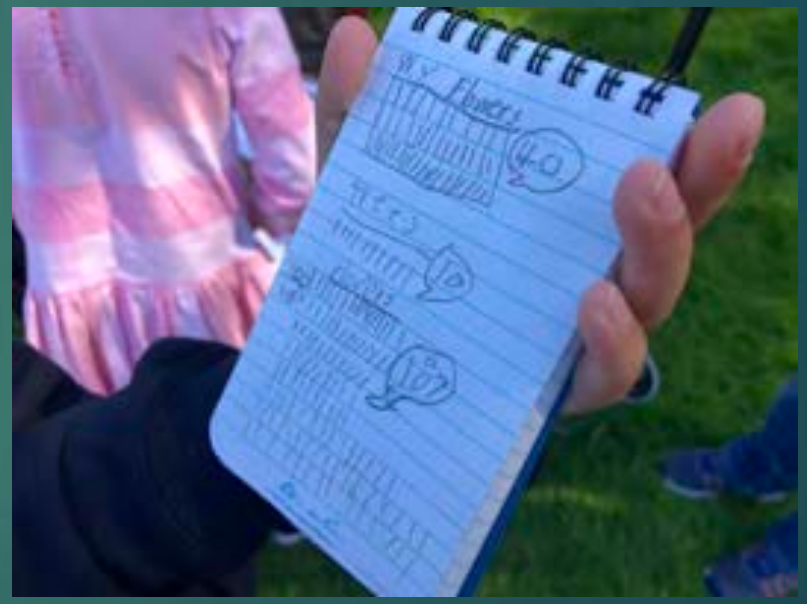
taking The Studio outdoors


Swirl by Swirl
Spirals in Nature
Joyce Sidman
Sarah C. Campb
Sarah C. Campbell
and Richard P. Campbell
Anywhere Artist
She creates art from anything, anytime!
Nikki Slade Robinson
Patchen Barss
Todd Stewart
Flow Spin Grow
Looking for Patterns in Nature

#100LeavesChallenge


HH


49 Sticks
53
70
49 Grass

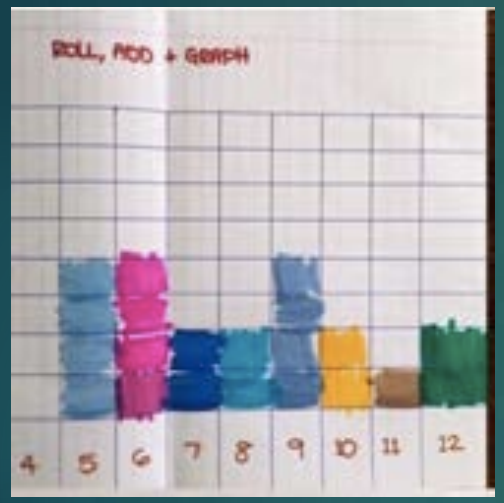


- What places and spaces might you find to play with mathematical ideas with your students?

considerations for remote learning

- weekly math plans
- outdoor math experiences
- at-home interdisciplinary projects
- Zoom math studio sessions

Weekly Math Plans



SD 38 K-12 Mathematics & Numeracy

Kindergarten – Grade Two: Week Twelve

Big Ideas: Number represents and describes quantity.
Computational fluency develops from a strong sense of number.

Curricular Content: counting, decomposing and representing quantities, addition and subtraction facts to 20, addition and subtraction to 20, 100 (concept, process)

Curricular Competencies: develop mental math strategies and abilities to make sense of quantities, develop, demonstrate, and apply mathematical understanding through play, inquiry and problem solving, engage in problem-solving experiences that are connected to place, story, community, and culture, communicate mathematical thinking in many ways, represent mathematical ideas in concrete, pictorial, and symbolic forms

Core Competencies focus: Positive Personal and Cultural Identity

Teachers and Families: The following are five problems/tasks to choose from for this week, based on the above curricular areas of focus.

Draw and write a math story for one of these questions: 2 and _____ make 7 / 9 + ___=12 / 28 + ___=33 / ___ + 20 =53 / 75 + ___ = 135 Choose numbers that stretch your thinking! What problem will you pose as part of your math story?
Choose three of these numbers: 10, 25, 89, 100, 142 What different ways can you represent the numbers? Consider using symbols, pictures, ten frames, tally marks, coins, etc.
Choose a number that stretches your thinking: 12, 25, 75, 99, 200 Find that quantity of items (rocks, seeds, books, blocks, toys). What different ways can you count the items? How can you show or record how you counted them?
Choose three questions that seem just right for you: 2 and 6, 8 and 4, 3 and 10, 9+8, 46+7, 52+9, 69+24, 98+37, 147+72 What different strategies can you use to solve these questions? What strategy/method do you think is a strength of yours?
Numeracy Task: There are 24 pieces of sidewalk chalk in a box. They need to be shared between four children. One child already has 3 pieces of chalk of their own. What are some different ways you could share the new box of sidewalk chalk? Which way do you think is the most fair and why? Use pictures, numbers and words to share your thinking. 

created by Janice Nowakowski for the Richmond School District
SD38/K-2MATH-12

RICHMOND
SCHOOL DISTRICT NO. 38

Outdoor Math Experiences

Outdoor Learning Experiences
for in-school and at-home learning
K-5 Mathematics & Numeracy
June 2020

Counting Collections
Find a collection of materials outside (leaves, rocks, twigs cones).
What different ways can you count them?
How could you record your counts?
Find plants, trees or flowers and think about how you could count them. For example, buttercups have five petals – find a patch of buttercups and count by 5s.

WODB
Using sidewalk chalk on concrete or twigs, create a 2x2 grid. Choose four numbers, shapes or natural items found outside and place one in each section.
How are they the same?
How are they different?
If you had to choose one thing to not belong, which would it be and why?
Leave your WODB out for others to think and talk about.

Math Walks
Go on a daily math walk, with a different focus each day. Take a notebook or clipboard with paper and pencil to record what you find.
What shapes do you see? What numbers can you find? What could we count? What strategies can you use to estimate the number of leaves on the tree? What do you notice about the different patterns you find in plants and trees?

created by Janice Novakowski for the Richmond School District
June 2020

RICHMOND
SCHOOL DISTRICT NO. 38

Patterning
Use materials found outside to create repeating patterns, growing patterns, patterns in circles or spirals.
Create patterns on safe concrete areas with sidewalk chalk, leaving parts for others to think and talk about.
What part of the pattern is missing?
What could come next?

Measuring
What could you find outside to measure?
Find 10 of something outside and put them in order from shortest to longest.
Choose an area outside (garden, field, backyard). *What different ways could you measure it? Length, width, area, perimeter.*
Record you measurements using pictures, numbers and words.

2D & 3D Shapes
Find twigs and use string to lash or tie them together to make nets/frames of 2D and 3D shapes.
How do shapes come together to make new shapes? What shapes do you see within your shape? What could you create with your shapes?

Tic Tac Toe
Gather materials outside to make a tic-tac-toe game or use chalk on a safe, clear concrete area.
What strategies do you use when playing tic-tac-toe? What other games could you create with materials you can find outdoors?

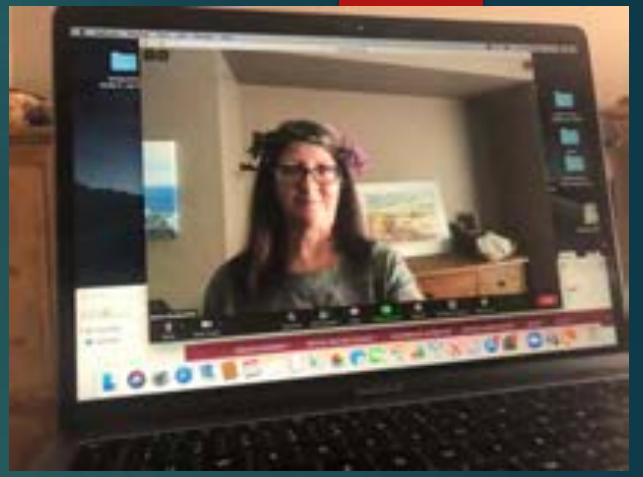
Suggested materials to pack in a outdoors "go bag": clipboard, paper, pencil, eraser, sidewalk chalk, string or yarn, scissors, ruler, measuring tape, magnifying glass.

created by Janice Novakowski for the Richmond School District
June 2020

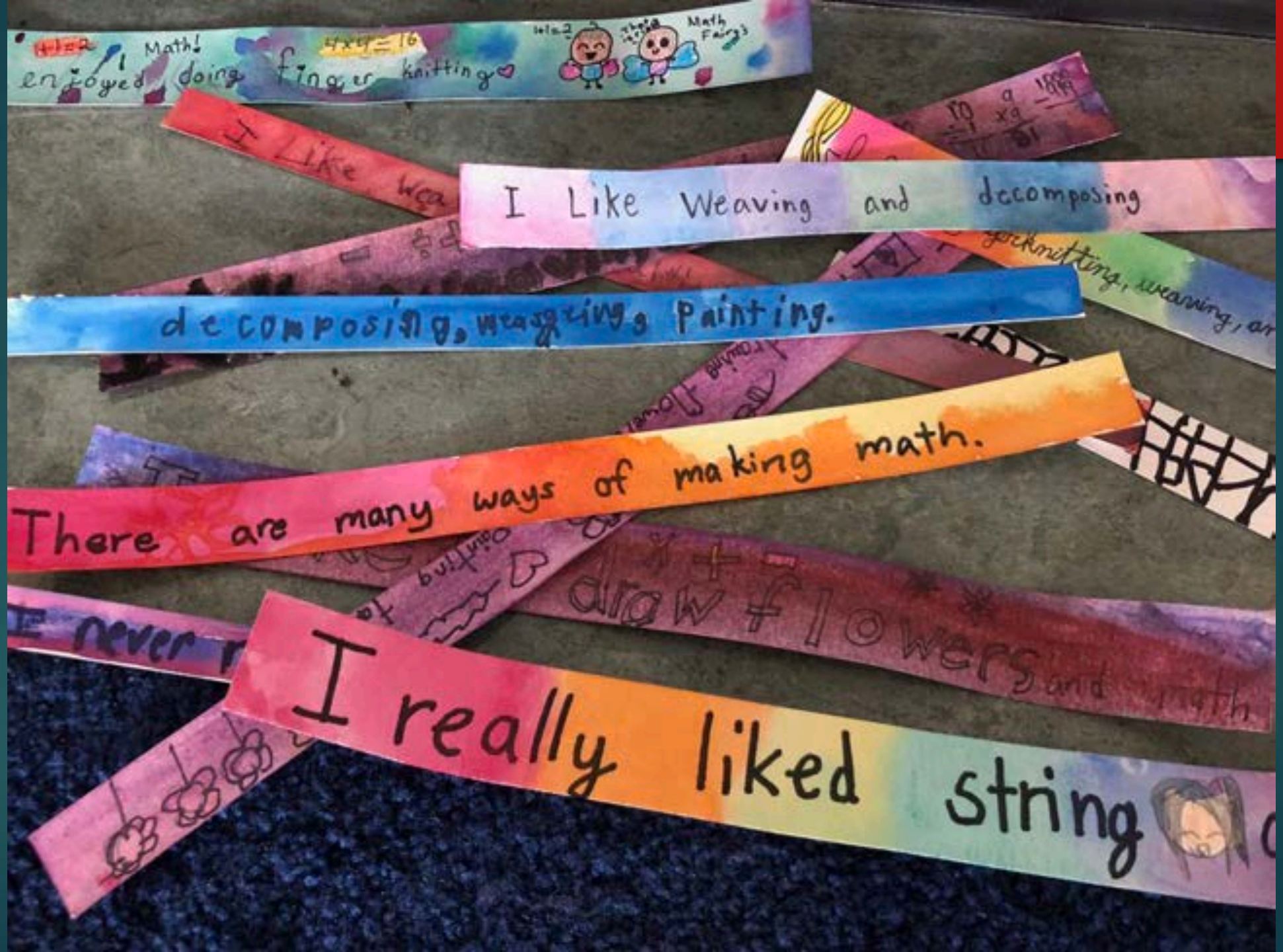
RICHMOND
SCHOOL DISTRICT NO. 38

At-Home Interdisciplinary Projects

Zoom math studio sessions



student reflections on The Studio experience


Math!
enjoyed doing finger knitting
I Like Weaving and decomposing
decomposing, weaving, Painting.
There are many ways of making math.
I really liked string


I realized how
Shapes, Patterns, Fun and more!
There are many ways of making math.
Paintings.
I really liked string art.


MATH IS Really Fun!
Math is beautiful

creating a new narrative for mathematics

- *"Math is beautiful."*
- *"Math is fun."*
- *"There are many ways of making math."*
- *"Math is everywhere."*
- *"We all do math."*

What math lives here?

uncovering mathematics in playful studio experiences

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jnovakowski